Transitions symmetries shape in $^{176-196}$Pt isotopes with interacting boson model

Heiyam Najy Hady
Kufa University/Education college for girl / Department of physics ,
Mohsin Kadhim muttaleb
BabylonUniversity/ Science college/Department of physics,

Abstract:
The symmetry states structure of $^{176-196}$Pt isotopes has been studied using the interacting boson model (IBM-1). The energy levels, the electromagnetic transitions probability $B(E2)$, the quadrupole moment of $2^+_1$ state and potential energy surfaces are analyzed which reveal the detailed nature of nuclei. In this chain nuclei evolve from harmonic vibrator to gamma soft rotor with wobble $a_0/\epsilon$ ratio ascent and descent in the first four isotopes then steady as straight line in the last six isotopes. The predicted theoretical calculations were compared with the experimental data in respective figures and tables, it was seen that the predicted results are in a good agreement with the experimental data. In the framework of IBM calculations (40 ) energy levels were determined for $^{176-196}$Pt isotopes. This investigation increases the theoretical Knowledge of all isotopes with respect to energy levels and reduced transition probabilities.

الخلاصة:
تمت دراسة تركيب الحالات المتماثلة لنظائر البلاتينيوم ($^{176-196}$) باستخدام نموذج البوسونات المتماثلة الأول. حددت مستويات الطاقة الاحتمالية للانتماءات الكهرومغناطيسية ($B(E2)$) ومخطط الانتقال العكسي للمستوى $2^+_1$، وسطح استقرار الجهد الذي يظهر الطبيعة المفصلة (gamma soft rotor) للنوى. في هذه السلسلة الراحلة تطور نظم التواكب الصلبة في الأربعة الأولى وفيها كخط مستقيم للنظام المتماثل. الحسابات النظرية المتوقعة قورنت مع البيانات العملية بجدول ورسوم خاصة IBM $^{176-196}$ Pt ويبدو أن النتائج المتوقعة متوافقة جيدًا مع البيانات العملية. في نطاق حسابات IBM (40) مستوي طاقة قد حدّد لنظام $^{176-196}$Pt، هذا البحث قد زاد المعرفة النظرية بجميع هذه النظائر بالنسبة لمستويات الطاقة الاحتمالية المتماثلة المختلطة.

Introduction:
The use of boson degree of freedom to describe the quantum dynamics of many fermion systems is a vast subject. The interacting boson model of Arima and Iachello has been successfully applied to a wide range of nuclear collective phenomena. The essential idea is that the low energy collective degrees of freedom in nuclei can be described by proton and neutron bosons with spins of 0 and 2. These collective building blocks interact. Different choices of $L=0$ (s-boson) and $L=2$ (d-boson) energies and interaction strengths give rise to different types of collective spectra. The IBM is a phenomenological model, that is to say its parameters are determined by fitting to the excitation spectra of nuclei. The interpretation of the boson as proton pairs and neutrons pairs is only manifested in the means by which $N_p$ and $N_n$ are chosen for a given nucleus. There is extensive literature that undertakes to interpret the bosons of the model microscopically (Arima et al,1976),(Casten et al,1988),(Wood et al,1992).

In the interacting boson model, collective excitations of nuclei are described by bosons. An appropriate formalism to describe the situation is provided by second quantization. One thus introduces boson creation (and annihilation) operators of multi polarity $l$ and $z$-component $m$. A boson model is specified by the number of bosons operators that are
introduced. In the interacting boson model -1 it is assumed that low-lying collective states of nuclei can be described in terms of a monopole bosons with angular momentum and parity \( j^r = 0^+ \) called \( s \) and a quadrupole boson with \( j^r = 2^+ \) called \( d \) (Iachello et al,1987). There are two basic concepts on which the IBM is based. One is that low-lying collective states in even-even nuclei can be described by only the valence nucleons, which form interacting fermion pairs. The other idea is that the fermion pairs couple to form bosons, carrying angular momentum \( (J) \). The energies \((\varepsilon_s \text{ and } \varepsilon_d)\), and the interactions of the \( s \) and \( d \) bosons, predict the low-lying excitations in the nucleus. There is 1 available magnetic substate for the \( s \) boson, determined by \((2J + 1)\), and 5 available magnetic substates for the \( d \) boson, forming a 6-dimensional space described by the group structure \((6)\). The quadrupole collectivity is a prominent aspect in the nuclear structure for both stable and exotic nuclei (Green,2009),(Nomura et al,2009). In 1997 W.Chou et. al. was carrying out calculation for 145 nuclei in the \( Z=50 \text{ - } 82 \) shell from \( A=120 \) to \( 200 \) with the IBM -1 using a constant set of procedures and a standardized set of six observables (Chou et al,1997). N. Pietralla et. al. was studying the excitation energy of \( 1^+ \) scissors mode and its empirical dependence on the nuclear deformation parameters ,the \(^{196}\text{Pt} \) isotopes was within this work (Pietralla,et al,1998). The Pt isotopes from \(^{188-196}\)Pt exhibit by N.Zamfir et.al.[4]a stable structure very close to \( O(6) \). In (2007) The systematics of \( g \) factor of the first excited \( 2^+ \) state vs neutron number \( N \) is studied by the projected shell model. The study covers the even-even nuclei of all isotopic chains from Gd to Pt. \( g \) factors are calculated by using the many-body wave functions that well reproduce the energy levels and \( B(E2) \) of the ground-state bands. For Gd to W isotopes the characteristic feature of the \( g \) factor data along an isotopic chain is described by the present model. Deficiency of the model in the \( g \) factor description for the heavier Os and Pt isotopes is discussed(Bao-An Bian et al,2007).

**Interacting Boson Model (IBM):**

The Lie algebra \( U(6) \) can be decomposed into a chain of sub algebras. If an appropriate chain of algebras can be found, the representations of each of these algebras can be used to label states with appropriate quantum numbers. This is because the states can be chosen that transform as the representations of each algebra. For applications to nuclei the chain of algebras must contain the subalgebra \( SU(3) \) since it is needed for states to have as a representation of the rotation group. In other words, \( SU(3) \) is required for states to have a good angular momentum quantum number. Three and only three chains of sub algebras have been found that contain the subalgebra \( SU(3) \). One of these chains is

\[
U(6) \ni U(5) \ni SU(5) \ni SU(3) \ni SU(2),
\]

Where under each algebra, the corresponding quantum number is given. Note that there are two quantum numbers given for the algebra \( SU(5) \). This is due to an ambiguity from reducing \( SU(5) \) to \( SU(3) \) and an additional quantum number is needed to uniquely specify the remaining representations. The quantum numbers \( L \) and \( M \) correspond to the angular momentum and magnetic quantum numbers (Ahn,2008).
where the $\alpha$ the Hamiltonian of Eq. (1) is given by Kirson and Dieperink, Schollton and Iachello of collective variables $\beta, \gamma$ (Puddu et al, 1980) in the first approach, one expresses complementary approaches are possible in discussing properties of collective spectra. In changes from nucleus to nucleus, giving rise to a large variety of collective spectra. Two consequence of the interplay between pairing and quadrupole correlations. This Hamiltonian is specified by 9 parameters, 2 appearing in the one body term, $\varepsilon_s, \varepsilon_d$ and $\gamma_0$ and 7 in the two body terms, $c_L (L = 0, 2, 4)$, $\tilde{u}_L (L = 0, 2)$ and $u_L (L = 0, 2)$. However, since the total number of boson (pairs) is conserved, $N = n_s + n_d$ (Arima et al, 1980).


$$ T^{(0)} = \alpha_s \delta_{s2} [d^+ s + s^+ d]_m^0 + \beta_2 [d^+ d]_m^0 + \gamma_0 \delta_{s0} [s^+ s]_m^0 \ldots \ldots (2) $$

Where $\alpha_2, \beta_1, \gamma_0$ are the coefficient of the various terms in the operator. This equation yields transition operators for E0, M1, E2, M3 and E4 transitions with appropriate value of the corresponding parameters.


$$ T^{(E2)} = \alpha_2 [d^+ s + s^+ d]_m^2 + \beta_2 [d^+ d]_m^2 \ldots \ldots (3) $$

It is clear that, for the E2 multipolarity, two parameters $\alpha_2$ and $\beta_2$ are needed in addition to wave function of the initial and final states.

The spectra of medium mass and heavy nuclei are characterized by the occurrence of low-lying collective quadrupole state. The actual way in which these spectra appear is consequence of the interplay between pairing and quadrupole correlations. This interplay changes from nucleus to nucleus, giving rise to a large variety of collective spectra. Two complementary approaches are possible in discussing properties of collective spectra. In the first approach, one expresses the collective Hamiltonian (and other operators) in terms of shape variables $\beta, \gamma$ (Puddu et al, 1980). The geometric properties of interacting boson model are particularly important since they allow one to relate this model to the description of collective states in nuclei by shape variables. It is more convenient to use in the discussion of the geometric properties of the interacting boson model another set of coherent states the projective states. These were introduced by Bore and Mottelson, Gnocchio and Kirson, Schollton and Dieperink, Schollton and Iachello, Ginocchio et al, 1980, (Dipernik et al, 1980, Bohr et al, 1980).

A general expression for this energy surface, as a function of $\beta$ and $\gamma$ state in term of the Hamiltonian of Eq. (1) is given by (Casten et al, 1988):

$$ E(N; \beta, \gamma) = \frac{N}{1 + \beta^2} \frac{\beta^2}{(1 + \beta^2)^2} \alpha \beta \cos 3 \gamma + \alpha \beta^2 + \alpha_4 \ldots \ldots (4) $$

where the $\alpha_4$’s are simply related to the coefficients of Eq. (1). One noted that $\gamma$ occurs only in the terms in $\cos 3 \gamma$, the energy surface has minima only at $\gamma = 0^\circ$ and $\gamma = 60^\circ$.

2164
Then the potential energy surface equation for the three symmetries can be given by the following equations (Iachello et al, 1987).

\[
E^{(1)}(N; \beta, \gamma) = E_0 + e_{0N} \frac{N \beta^2}{1 + \beta^2} + f_{1N} (N-1) \frac{\beta^4}{(1 + \beta^2)^2}
\]

\[
E^{(a)}(N; \beta, \gamma) = E_0 - k^2 \left[ \frac{N}{(1 + \beta^2)} (5 + \frac{11}{4} \beta^2) + \frac{N(N-1)}{(1 + \beta^2)^2} \left( \frac{\beta^4}{2} + 2\sqrt{2} \beta^3 \cos 3\gamma + 4\beta^3 \right) \right] \ldots (5)
\]

\[
E^{(a1)}(N; \beta, \gamma) = E_0 + (2B + 6C) \frac{A}{4} N(N-1) \frac{1 - \beta^2}{1 + \beta^2}
\]

Calculations and results:

Calculations of energy levels for even-even \(^{176-196}\)Pt isotopes were performed with the whole Hamiltonian (eq.1) using IBM-1 computer code. For \(^{176-196}\)Pt nuclei (Z=78) have (10-12 bosons where N< 104 and 12-6 bosons where N> 104) formed (2 proton hole) bosons and (10-12) neutron particle bosons and (12-6) neutron hole bosons.

The parameters of equation (1) were calculated from the experimental schemes of these nuclei [19-28] and the analytical solutions for the three dynamical systems see reference (Casten et al, 1988). These parameters were tabulated in table (1). The calculated and experimental energy levels and the parameters value are exhibit in figure (2).

The calculations of B(E2) values were performed using computer code “IBMT”. The parameters in E2 operator eq.(3) were determined by fitting the experimental B(E2; 2\(^+_i \rightarrow 0\(^+_j\)) data [19-28], and the parameters were listed in table (1) and (2), where

\[
\beta_i = \frac{-0.7}{5} \alpha_3, \gamma = \sqrt{7} \alpha_2 \text{ and } \alpha = 0 \text{ And } E2SD = \alpha_2, E2DD = \sqrt{5} \beta_2
\]

in SU(5), SU(3) and O(6) respectively [4-7]. The converter coefficient between \((e^2b^2)\) and \((W.u)\) is \(B(E2)_{W.u} = \frac{B(E2)e^2b^2}{5.943 \times 10^{-4} A^{1/3} e^{-2} b^2} \).

The values of the parameters which gave the best fit to experimental [19-29] are given in table (1). The parameters of the energy surface were calculated by transforming the parameters of Hamiltonian of equation 1 by several equations see reference (Casten et al, 1988), and they are found to be as in table (1) to draw the energy functional \(E(N; \beta, \gamma)\) as a function of \(\beta\) and the contour plots in the \(\gamma-\beta\) plane fig.(3).
Table (1) The parameters of the Hamiltonian equation, The parameters obtained from the programs IBMP code for potential energy surface and E2 operators used for the description of the $^{176-196}$Pt isotopes.

<table>
<thead>
<tr>
<th>Isotope</th>
<th>N</th>
<th>$\varepsilon$</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$e_1$</th>
<th>$e_2$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>$\alpha_4$</th>
<th>E2SD</th>
<th>E2DD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{178}$Pt</td>
<td>10</td>
<td>0.185</td>
<td>0.0</td>
<td>0.001</td>
<td>0.0</td>
<td>0.584</td>
<td>0.065</td>
<td>0.0</td>
<td>0.0</td>
<td>0.226</td>
<td>-0.165</td>
<td>-0.165</td>
<td>-0.165</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$^{178}$Pt</td>
<td>11</td>
<td>0.06</td>
<td>0.24</td>
<td>0.039</td>
<td>0.0</td>
<td>0.08</td>
<td>0.0</td>
<td>0.0</td>
<td>0.41</td>
<td>0.06</td>
<td>0.0</td>
<td>0.0</td>
<td>0.28</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>$^{180}$Pt</td>
<td>12</td>
<td>0.195</td>
<td>0.134</td>
<td>0.02</td>
<td>0.0</td>
<td>0.023</td>
<td>0.0</td>
<td>0.0</td>
<td>0.35</td>
<td>0.034</td>
<td>0.0</td>
<td>0.0</td>
<td>0.272</td>
<td>-0.19</td>
<td></td>
</tr>
<tr>
<td>$^{184}$Pt</td>
<td>12</td>
<td>0.19</td>
<td>0.146</td>
<td>0.021</td>
<td>0.0</td>
<td>0.028</td>
<td>0.0</td>
<td>0.0</td>
<td>0.36</td>
<td>0.037</td>
<td>0.0</td>
<td>0.0</td>
<td>0.297</td>
<td>-0.18</td>
<td></td>
</tr>
<tr>
<td>$^{186}$Pt</td>
<td>11</td>
<td>0.354</td>
<td>0.034</td>
<td>0.001</td>
<td>0.0</td>
<td>0.0001</td>
<td>0.0</td>
<td>0.0</td>
<td>0.36</td>
<td>0.009</td>
<td>0.0</td>
<td>0.0</td>
<td>0.232</td>
<td>-0.162</td>
<td></td>
</tr>
<tr>
<td>$^{188}$Pt</td>
<td>10</td>
<td>0.759</td>
<td>0.12</td>
<td>0.0077</td>
<td>0.0</td>
<td>0.0001</td>
<td>0.0</td>
<td>0.0</td>
<td>0.806</td>
<td>0.03</td>
<td>0.0</td>
<td>0.0</td>
<td>0.228</td>
<td>-0.159</td>
<td></td>
</tr>
<tr>
<td>$^{190}$Pt</td>
<td>9</td>
<td>0.76</td>
<td>0.12</td>
<td>0.0025</td>
<td>0.0</td>
<td>0.0001</td>
<td>0.0</td>
<td>0.0</td>
<td>0.776</td>
<td>0.03</td>
<td>0.0</td>
<td>0.0</td>
<td>0.197</td>
<td>-0.138</td>
<td></td>
</tr>
<tr>
<td>$^{192}$Pt</td>
<td>8</td>
<td>0.726</td>
<td>0.175</td>
<td>0.0027</td>
<td>0.0</td>
<td>0.0001</td>
<td>0.0</td>
<td>0.0</td>
<td>0.751</td>
<td>0.044</td>
<td>0.0</td>
<td>0.0</td>
<td>0.127</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>$^{194}$Pt</td>
<td>7</td>
<td>0.695</td>
<td>0.175</td>
<td>0.0002</td>
<td>0.0</td>
<td>0.0001</td>
<td>0.0</td>
<td>0.0</td>
<td>0.705</td>
<td>0.044</td>
<td>0.0</td>
<td>0.0</td>
<td>0.146</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>$^{196}$Pt</td>
<td>6</td>
<td>0.578</td>
<td>0.175</td>
<td>0.0105</td>
<td>0.0</td>
<td>0.0001</td>
<td>0.0</td>
<td>0.0</td>
<td>0.65</td>
<td>0.044</td>
<td>0.0</td>
<td>0.0</td>
<td>0.152</td>
<td>0.0</td>
<td></td>
</tr>
</tbody>
</table>

Table (2) Comparison between present values of $\text{B(E2)}$ (in unit $e^2b^2$) for even-even $^{176-196}$Pt isotopes (Theo.) and experimental ones (Exp.) (Basunia,2006),(Achterberg et al.2009), (Wu et al.2003), (Baglin,2010), (Baglin,2003), (Singh,2002), (Singh,2003), (Coral et al.1998), (Singh,2006), (Xiaolong,2007). The quadrupole moment of $2_1^+$ state listed in last line.

<table>
<thead>
<tr>
<th>Transitions</th>
<th>$2_1^+\rightarrow0_1^+$</th>
<th>$2_1^+\rightarrow0_2^+$</th>
<th>$2_1^+\rightarrow2_1^+$</th>
<th>$4_1^+\rightarrow2_1^+$</th>
<th>$Q_2^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isotope</td>
<td>Th.</td>
<td>Exp.</td>
<td>Th.</td>
<td>Exp.</td>
<td>Th.</td>
</tr>
<tr>
<td>$^{178}$Pt</td>
<td>0.55</td>
<td>0.506</td>
<td>0.0</td>
<td>-</td>
<td>1.007</td>
</tr>
<tr>
<td>$^{180}$Pt</td>
<td>0.57</td>
<td>0.081</td>
<td>0.0</td>
<td>-</td>
<td>0.992</td>
</tr>
<tr>
<td>$^{182}$Pt</td>
<td>0.8</td>
<td>0.698</td>
<td>0.55</td>
<td>-</td>
<td>1.34</td>
</tr>
<tr>
<td>$^{184}$Pt</td>
<td>0.59</td>
<td>0.59</td>
<td>0.0</td>
<td>-</td>
<td>1.08</td>
</tr>
<tr>
<td>$^{186}$Pt</td>
<td>0.519</td>
<td>0.518</td>
<td>0.0</td>
<td>-</td>
<td>0.93</td>
</tr>
<tr>
<td>$^{188}$Pt</td>
<td>0.35</td>
<td>0.378</td>
<td>0.0</td>
<td>-</td>
<td>0.62</td>
</tr>
<tr>
<td>$^{190}$Pt</td>
<td>0.27</td>
<td>0.382</td>
<td>0.0</td>
<td>-</td>
<td>0.37</td>
</tr>
<tr>
<td>$^{192}$Pt</td>
<td>0.28</td>
<td>0.328</td>
<td>0.0</td>
<td>0.004</td>
<td>0.38</td>
</tr>
<tr>
<td>$^{194}$Pt</td>
<td>0.24</td>
<td>0.274</td>
<td>0.0</td>
<td>-</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Discussion and conclusion:

The study of phase transitions is one of the most exciting topics in Physics it has been in fact argued that moving from the spherical to the unstable deformed case within the IBM. The energy ratio $\text{E(Jiyor)/E(2ir)}$ for the $J_1^+=4_i^+6_i^+$ and $8_i^+$ levels for the doubly even platinum isotopes with both the vibtational and non axial gamma-soft rotor limit for this ratio were shown on the figure (4).The behavior of the ratio of the energies of the first $4_i^+$ and $2_i^+$ states were good criterion for the shape transition. The value of $R_{4/2}$ ratio has the limiting value (2) for a quadrupole vibrator (2.5) for a non axial gamma-soft rotor as can seen in the figure (4) it ceases gradually from about 2.13 to about 2.5, the agreement between the calculated result show that $R_{4/2}$ tend to 2.5 for all Pt isotopes as well as $R_{6/2}$ variety from (3.42 to 4.3) and $R_{8/2}$
(from 4.9 to 6.5) which ensure this trend where typical value of $R_{6/2}$ and $R_{8/2}$ were (3 and 4, 4.5 and 7) for US(5) and O(6) respectively which mean that their structure seem to be varying from harmonic vibrator to gamma soft rotor.

A successful nuclear model must yield a good description of not only the energy spectrum of the nucleus, but also of its electromagnetic properties. The comparison between experimental and IBM expectation of $B(E2)$ transitions for $(2_{1}^{+}\rightarrow 0_{1}^{+})$, $(2_{2}^{+}\rightarrow 0_{1}^{+})$, $(2_{2}^{+}\rightarrow 2_{1}^{+})$ and $(4_{1}^{+}\rightarrow 2_{1}^{+})$ in table (2) were acceptable values.

The potential surface in $^{176-196}$Pt was different from a spherical vibrator which minimum at $\beta=0$ and have circular contours centered at this point. The contours resemble those of a SU(5)→O(6) transition region potential since the minimum potential occurs approximately at $\beta=0.2$ which lies between $\beta=0$ for SU(5) and $\beta=1$ for O(6) see fig.(3) finally the $Q2_{1}^{+}$ value make clear the similarity with SU(5)→O(6) transitions region. The slightly gradation in the $^{176-196}$Pt nuclei behavior can be interpreted if we look at the neutron distribution in the nuclear shells, the $^{176,178}$Pt nuclei occupy $2f_{7/2}$, $^{180,184}$Pt in the $2f_{5/2}$, $^{186,188}$Pt in $3p_{3/2}$, $^{190}$Pt in $3p_{1/2}$ while $^{192-196}$Pt in $1i_{13/2}$ sub levels respectively.

In the framework of IBM calculations (40) energy levels were determined for $^{176-196}$Pt isotopes as $^{2+}_{2}:0.52$ MeV, $^{2+}_{3}:0.72$MeV, $^{3+}_{1}:0.38$MeV, $^{4+}_{2}:0.63$MeV, $^{4+}_{3}:0.79$MeV, $^{5+}_{1}:0.93$MeV and $^{6+}_{2}:1.33$ MeV) for $^{176}$Pt, $(2^{+}_{3}:0.41$MeV, $^{3+}_{1}:0.83$MeV, $^{4+}_{2}:0.86$MeV, $^{4+}_{3}:1.06$MeV, $^{5+}_{1}:0.93$ MeV and $^{6+}_{2}:1.33$ MeV) for $^{178}$Pt, $(3^{+}_{1}:0.218$MeV, $^{4+}_{2}:0.408$MeV, $^{5+}_{1}:0.59$MeV and $^{6+}_{2}:0.76$ MeV) for $^{180}$Pt, $(3^{+}_{1}:0.228$MeV, $^{4+}_{2}:0.42$MeV, $^{4+}_{3}:0.432$MeV, $^{5+}_{1}:0.61$MeV and $^{6+}_{2}:0.8$ MeV) for $^{184}$Pt, $(5^{+}_{1}:0.62$ MeV and $^{6+}_{2}:0.88$ MeV) for $^{186}$Pt, $(3^{+}_{1}:0.93$ MeV, $^{5+}_{1}:1.46$ MeV and $^{6+}_{2}:1.56$ MeV) for $^{188}$Pt, $(4^{+}_{2}:1.07$ MeV, $^{4+}_{3}:1.53$ MeV, $^{5+}_{1}:1.55$ MeV and $^{6+}_{2}:1.58$ MeV) for $^{190}$Pt, $(2^{+}_{3}:1.76$MeV, $^{4+}_{3}:1.79$MeV, $^{5+}_{1}:1.81$MeV and $^{6+}_{2}:1.84$ MeV) for $^{192}$Pt, $(4^{+}_{3}:1.87$MeV, $^{5+}_{1}:1.8$MeV and $^{6+}_{2}:1.25$ MeV) for $^{194}$Pt, and $(5^{+}_{1}:1.9$MeV and $^{8+}_{1}:2.36$ MeV) for $^{196}$Pt, see fig.(2).

This investigation increases the theoretical knowledge of all isotopes with respect to energy levels and reduced transition probabilities. Its concluded that more experimental data were required to fully investigate the level structure of these nuclei.
Fig. (1): The values of the parameters (ε, a₀, a₁ and a₂ / ε) were calculated from the experimental schemes ¹⁷⁰⁶(Basunia, 2006), (Achterberg et al, 2009), (Wu et al, 2003), (Baglin, 2010), (baglin, 2003), (Singh, 2002), (Singh, 2003), (Coral et al, 1998), (Singh, 2006), (Xiaolong, 2007) of ¹⁷⁰⁶Pt isotopes.
Fig. (2): A comparison between theoretical values of energy levels and the corresponding experimental one for $^{176}$Pt (Basunia, 2006), (Achterberg et al, 2009), (Wu et al, 2003), (Baglin, 2010), (Baglin, 2003), (Singh, 2002), (Singh, 2003), (Coral et al, 1998), (Singh, 2006), (Xiaolong, 2007).
Fig. (3): The energy functional $E(N; \beta, \gamma)$ as a function of $\beta$ and the corresponding $\beta$-$\gamma$ plot for $^{176-196}$Pt isotopes.

Fig. (4): Calculated and Experimental ratios (Basunia, 2006), (Achterberg et al., 2009), (Wu et al., 2003), (Baglin, 2010), (Baglin, 2003), (Singh, 2002), (Singh, 2003), (Coral et al., 1998), (Singh, 2006), (Xiaolong, 2007). $4^{+}/2^+$, $6^{+}/2^+$, and $8^{+}/2^+$ for $^{176-196}$Pt isotopes.

References:
Bao-An Bian, Yao-Min Di, Gui-Lu Long, Jing-ye Zhang, and Javid A. Sheikh,”
Systematics of g factors of 21+ states in even-even nuclei from Gd to Pt: A
microscopic description by the projected shell model, Phys. Rev. C 75, 014312
(2007).
Baglin C.,Nuclear Data Sheets,111,275 (2010).
Casten R., von Brentano P.and Haque A.,”Evidence for an underlying SU(3) structure
Chou W., Zamfir N.and Casten R.,”Unified description of collective nuclei with the
Green K., “Nuclear structure of 112Cd through studies of β decay” ,Master thesis , The
University of Guelph, 2009.
Ginocchio J. N. and Kirson M. W.” relationship between the Bohr collective
Iachello F.and Arima A.,”The interacting boson model “, Cambridge University Press ,
1987.
Nomura K., Shimizu N., Otsuka T.,”new formulation of interacting boson model and
National Nuclear Data Center, Brookhaven National Laboratory,
Pietralla N., von Brentano P., Herzberg R., Kneissl U., Judice N., Maser H., Pitz H.and
Zilges A., “Systematics of the excitation energy of the 1+ scissors mode and its
Puddu G.and Scholten O.,”Collective quadrupole state of Xe ,Ba, and Ce In the
Singh B ,,Nuclear Data Sheets, 95, 387 (2002).
Wood J., Hayde K., Nazarewicz W., Huyse M. and Van Duppen P.,”Coexistence in even
Wu S.and Niu H.,Nuclear Data Sheets ,100, 483 (2003).