The Impact of the Fluid Layer Thickness on the Onset of Transient Convection in Porous Media

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Abstract
The satisfactory performance of porous media as insulation in industrial applications depends upon the thermal properties of both the fluid and solid phases. Simulation of macroscopic of the onset of transient convection in porous media was proposed for a semi-infinite saturated porous media (glass-water) with heating from below as a boundary condition.

Effective thermal properties have been affected directly in the calculation of macroscopic model of heat transfer rate. Through this model was simulated the onset natural convection by calculated the transient convection of heat transfer coefficient between the top surface layer of solid phase and the bulk temperature of fluid phase by two values of heat flux at different void space of fluid layer. Also, Nusselt number and Rayleigh number of porous media were calculated at different interval times of heating.

Key words: porous media, onset convection, transient convection of porous media, saturated porous media.

1. Introduction
Heat transfer with a porous media occurs due to temperature gradients in the fluid and solid phases, and depends on the thermal properties of both the fluid and solid. Natural convection heat transfer in a saturated porous media have wide range of applications in engineering such as solar collector, insulation for building, cooling of electronic devices as PC, TV, etc. Those applications were reviewed by (Yasin et al., 2006).

Thermal properties of porous media are the main factor affecting the heat transfer rate from or to it. Thus the study of these properties gives a good guide to understanding the mechanism of heat transfer in void, solid and pure fluid regions. Variation of thermal properties affects directly in the calculation of heat transfer rate. Due to the composite structure of porous media, there are effective thermal properties of it. These effective thermal properties such as effective thermal conductivity, effective thermal diffusivity, and effective heat capacity have been the topic of many researches branches in physics, engineering, and chemistry.

(Tan and Thorpe, 1996) showed the thermal instability criterion for the onset of buoyancy convection based on an adverse linear temperature gradient in a fluid layer. For natural convection induced by a time-dependent and non-linear temperature profile, they developed a new transient Rayleigh number for the deep fluid for various boundary conditions. They pointed that, the knowledge of natural convection in a saturated porous
media is of considerable interest because of its importance in modeling of the heat transfer of geothermal reservoir and engineering applications, which include the high performance insulation for building and cold storage.

(Tan and Sam, 1999) showed the theory of transient convection in porous media was proposed for a semi-infinite saturated porous media with FST (fixed surface temperature) boundary condition. A computation fluid dynamic (CFD) package FLUENT was used to simulate the onset of convection in porous media. Transient Rayleigh number for unsteady-state heat conduction was defined. They found the Nusselt number in the range between 2.5 and 3.0, depending on the rate of heat transfer.

In the present work assume glass-water porous media as a sample for analysis with different void space of fluid layer. The microstructure of glass-water sample was assumed that the fluid and solid phases in parallel configuration as shown in figure (1). Macroscopic model of transient heat transfer was predicted. This model was simulated the temperature profile of temperature difference between the top surface layer of solid phase and the bulk temperature of the fluid phases through heating the porous media from the down by two values of heat flux. Also, the Nusselt number and Rayleigh number were calculated at different interval times of heating.

2. Mathematical Analysis

The use of macroscopic variables leads to the definition of effective properties which empirically account for the effect of the microstructure. Much theoretical development has been devoted to the modeling of the microscopic processes in order to predict the effective macroscopic properties.

When a macroscopic description is used to model conduction heat transfer in porous media, the effective thermal conductivity is defined by (Deissler and Boegli, 1985) as:

\[ k_e = \phi k_f + (1 - \phi) k_s \]  

Typically the effective thermal conductivity is assumed isotropic. Also, the effective volumetric heat capacity is a macroscopic extensive property, which was defined by (Batchelor and O'Brien, 1977) as:

\[ (\rho c_p)_e = \phi (\rho c_p)_f + (1 - \phi) (\rho c_p)_s \]  

Where \( \phi \) is the porosity. It is defined as the fraction of the void space volume to total volume.

Heat conduct within the porous medium can be described by (Nozad et al, 1985) as:
Temperature, on the other hand is an intensive macroscopic property. Therefore, in addition to being a function of the fluid and solid phase thermal conductives, $k_e$ is also a function of microstructure of the porous medium. Further, if the porous medium is saturated by water, i.e. the void space is completely occupied by the fluid phase, then buoyancy induced convective currents may develop within the void space and enhance the macroscopic effective thermal conductivity. This was detailed by (Hans, 1999).

To investigate onset convection in saturated porous medium heated from below. Hence, it has been investigated a saturated porous layer subjected to a vertical temperature gradient. Using Darcy flow model to simulate the onset of convection as assuming local thermal equilibrium between the solid and fluid phase.

### 2.1. Numerical Analysis of Heat Transfer Model

Heat conduction for semi-infinite medium subjected to heat flux has been fixed below the sample. Heat equation has been used to describe conduction within two-phase of porous medium, heated with rise time, at different values of heat flux.

It is important to determine the temperature distribution within the depth direction of macroscopic layer of porous medium heated by heat flux.

Assuming the thermal effective properties are independent of temperature, a one dimensional heat conduction equation under penetrating heat flux is represented as:

$$\frac{1}{\alpha_e} \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial y^2}$$

(4)

Where $\alpha_e$ is the effective thermal diffusivity of porous media which is

$$\alpha_e = \frac{k_e}{(\rho c_p)_e}$$

(5)

Solving equation (4) according to the following the initial and boundary conditions for macroscopic structure as shown in figure (2).

- The initial condition $T(y, 0) = \theta_0$
- The boundary condition $y = 0, -k_e \frac{\partial T}{\partial y} = q$

A numerical solution of equation (4) depends upon the finite difference method explicit technique to calculate the interior temperature through y-axis of porous medium layer macroscopic structure. The indices $j$ will be used for indicating the points along y-direction and index $n$ will be used for indicating the points over the time layers. The distances between the points in the established grid are $\Delta y$ and $\Delta t$. The grid is represented by figure (2). Notice that the temperature values in the time layer $n$ are known, and that we search for temperature values at the time $n+1$. The finite difference form of the equation (4) with explicit formulation is:

$$\frac{1}{\alpha_e} \frac{T^n_{lj+1} - T^n_{lj}}{\Delta t} = \frac{T^n_{lj+1} - 2T^n_{lj} + T^n_{lj-1}}{\Delta y^2}$$

(6)

Typical explicit approximation has been explained. The principles of stability analysis with numerical schemes are discussed by (Anderson et al., 1984), where

$$= \left( \alpha_e \frac{\Delta t}{\Delta y^2} \right)$$

is the convergence factor.

Then equation (6) becomes

\[ \frac{1}{\alpha_e} \frac{T^n_{lj+1} - T^n_{lj}}{\Delta t} = \frac{T^n_{lj+1} - 2T^n_{lj} + T^n_{lj-1}}{\Delta y^2} \]

(6)
The above initial and boundary conditions were subjected in order to solve equation (7):

\[ T_{i}(i,j)(n + 1) = (T_{i}(i, j + 1)n + (1 - 2^2 \cdot \gamma) T_{i}(i, j))n + (T_{i}(i, j - 1)n) \]

For ab, let us perform an energy balance on an arbitrary nodes at y=0, on face ab, surrounded by nodes a, b, and 2 as shown in figure (2). The expression for the heat conducted from the internal node 2, will be similar to the ones developed for heat conduction of internal nodes.

The temperature distribution within half of sample as shown in figure (3)

At point 1, x=0, y=0, and \( \Delta x=\Delta y \), then the energy balance as :

\[ T_{1}(i,j)(n + 1) = 2(T_{1}(i, j)+ (1 - 2^2 \cdot \gamma) T_{1}(i, j)) + 4\gamma \Delta y/\kappa_{1} \]  

When y \( \geq 0 \), heat conduction through penetrating macroscopic structure of porous medium as shown in figure (3), then equation (6) becomes:

\[ T_{2}(i,j)(n + 1) = (T_{2}(i, j+ 1)n + (1 - 2^2 \cdot \gamma) T_{2}(i, j))n + (T_{2}(i, j- 1)n) \]
Through simulation program for equations (8) and (9) calculated the surface
temperature of solid phase and the bulk temperature of fluid phase for every interval rise
time. It has been investigated through the above analysis the onset heat transfer
coefficient as for every interval time by Newton law as:

\[ h = \frac{Q}{A \Delta T} \]  
(10)

where

\[ \Delta T = T_{\text{surface of solid phase}} - T_{\text{bulk of fluid}} \]

When to focus upon natural convection alone, then the porous medium would be
modeled as a continuum having macroscopic properties, including a stagnant effective
thermal conductivity. Hence the parameter of interest would then be a Nusselt number
based upon the stagnant effective thermal conductivity, that is

\[ \text{Nu}_e = \frac{h L}{\bar{\kappa}_s} \]  
(11)

Where \( \text{Nu}_e \) appears to be appropriate parameter to describe the macroscopic effective
thermal conductivity in system where a natural convection is present. Some
investigations for example, base the Nusselt number upon the fluid thermal conductivity
as

\[ \text{Nu}_f = \frac{h L}{\bar{\kappa}_f} \]  
(12)

(Nield and Bejan, 1999) showed the Rayleigh number of fluid as

\[ \text{Ra}_f = \frac{\rho_f g \beta L^3 \Delta T}{\bar{\alpha}_f} \]  
(13)

They investigated natural convection in saturated porous medium heated from below;
they consider the porous medium analog of Rayleigh number for single phase internal
natural convection. The basic problem is one of a fluid layer which is infinite in extent
and heated from below.

Due assuming a local thermal equilibrium (Nield and Bejan, 1999)
approximation, they perform a linear stability analysis using the Darcy flow model to
predict the onset of convection currents. They considered that the onset of convection
occurs when \( \text{Ra}_m \geq 4\pi^2 \), where \( \text{Ra}_m \) is the porous medium Rayleigh number, which is
defined as

\[ \text{Ra}_m = \text{Ra}_e \cdot \text{Da} \]  
(14)

Da is the Darcy number and presented as

\[ \text{Da} = \frac{K}{H^2} \]  
(15)

Where \( H \) is the layer thickness and \( K \) is the permeability. (TAN and SAM, 1999)
showed the permeability of various sizes of beads which were obtained by using the
Kozeny-Carmen relation

\[ \frac{K}{172 \cdot 8 \phi^3} \left(1 - \phi \right)^2 \]  
(16)
in which \( d \) was the diameter of the beads and \( \phi \) the porosity.
3. Results and Discussion

In the present work, it has successfully to design program in order to simulate the onset transient convection in porous media. Table (1) shows the values of thermal properties for glass and water at $T_0 = 25^\circ C$. It has been needed to use the equation 1 to 5 in order to calculate the effective thermal prosperities of porous media (glass-water) as tabulated in table (1).

**Table (1) values of thermal properties and effective thermal properties of (glass-water).**

<table>
<thead>
<tr>
<th>Thermal properties</th>
<th>Effective thermal properties</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho$ (kg/m$^3$)</td>
</tr>
<tr>
<td>Solid phase</td>
<td>2200</td>
</tr>
<tr>
<td>(glass)</td>
<td></td>
</tr>
<tr>
<td>Fluid phase</td>
<td>995.7</td>
</tr>
<tr>
<td>(water)</td>
<td></td>
</tr>
</tbody>
</table>

It has been pointed some of these results in tables (2) and (3) as example to show some results of fixed value of heating time. These tables illustrate the values of different parameters were calculated through the simulation program of porous media (glass-water) at time 120 seconds and with two assumption values of heat flux 400W/m$^2$ and 800W/m$^2$. These parameters were determined for different void space of fluid layer 1, 2, and 3mm respectively.

**Table (2) values of different parameters calculated from simulation of glass-water with heat flux 400 W/m$^2$ and at time 120 seconds.**

<table>
<thead>
<tr>
<th>Fluid bead size</th>
<th>$K$</th>
<th>Da</th>
<th>$\Delta T$ (°C)</th>
<th>$h$ (W/m$^2$ °C)</th>
<th>Nu$_f$</th>
<th>Nu$_e$</th>
<th>Ra$_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 mm</td>
<td>8.920×10$^{-10}$</td>
<td>8.926×10$^{-4}$</td>
<td>2.643</td>
<td>151.331</td>
<td>12.653</td>
<td>10.851</td>
<td>72.783</td>
</tr>
<tr>
<td>2 mm</td>
<td>3.570×10$^{-9}$</td>
<td>8.924×10$^{-4}$</td>
<td>7.475</td>
<td>53.508</td>
<td>4.473</td>
<td>3.836</td>
<td>572.931</td>
</tr>
<tr>
<td>3 mm</td>
<td>8.033×10$^{-9}$</td>
<td>8.925×10$^{-4}$</td>
<td>6.364</td>
<td>52.848</td>
<td>5.254</td>
<td>4.506</td>
<td>1946.563</td>
</tr>
</tbody>
</table>

**Table (3) values of different parameters calculated from simulation of glass-water with heat flux 800 W/m$^2$ and at time 120 seconds.**

<table>
<thead>
<tr>
<th>Fluid bead size</th>
<th>$K$</th>
<th>Da</th>
<th>$\Delta T$ (°C)</th>
<th>$h$ (W/m$^2$ °C)</th>
<th>Nu$_f$</th>
<th>Nu$_e$</th>
<th>Ra$_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 mm</td>
<td>8.920×10$^{-10}$</td>
<td>8.926×10$^{-4}$</td>
<td>2.848</td>
<td>280.850</td>
<td>23.482</td>
<td>20.138</td>
<td>72.741</td>
</tr>
<tr>
<td>2 mm</td>
<td>3.570×10$^{-9}$</td>
<td>8.924×10$^{-4}$</td>
<td>12.542</td>
<td>63.785</td>
<td>5.333</td>
<td>4.573</td>
<td>561.583</td>
</tr>
</tbody>
</table>

It has been shown the permeability and Darcy number were calculated for different void space of fluid layer by using two equations (15) and (16), which were dependent upon the void space dimensions and the value of porosity of glass-water media, this value was about 0.387.

The other parameters such as Nusselt number of porous media (Nu$_e$), which was dependent upon the macroscopic effective thermal conductivity of two phases (solid-fluid). It has been illustrated, that the Nu$_e$ decreased with increasing the void space of fluid layer phase. Also, the Nusselt number of fluid phase Nu$_f$, which was dependent upon the thermal conductivity of fluid only, has approximately the same relation and response of Nu$_e$. That was affected by the time interval of heat and the values of heat...
supply. Other hand, it has shown the value of onset of heat transfer coefficient, which was high when the void space is small and decreasing with large value of a void space fluid layer and effected with values of heat flux. It has been noted the Rayleigh number of porous media (Ra_m), which was dependent upon the Darcy number. The values of Rayleigh number were calculated in range 72 for 1mm void space, and 1947 for 3mm void space.

Figures (4) and (5) show the temporal variation of heat transfer coefficient for three values of void space of fluid layer with two values of heat flux. It has been shown at void space of 1mm the value of the onset of heat transfer coefficient was increased with time of heat, and with increasing the value of heat flux that more signally with 800W/m^2. This is due to the heat transfer rate is higher in small void space of fluid layer phase. While, it has been shown at heat flux 400W/m^2 the heat transfer coefficient decreased after time 120 seconds. This is due to the behavior of porous media material through heating process. While, it was pointed lower value of onset heat transfer coefficient when void space of fluid layer increasing and with increasing the time of heating. At two values of heat flux the lower value of heat transfer coefficient was calculated at a void space of 2mm. This is due the value of difference temperature between the surface of solid phase and the bulk temperature of fluid phase ΔT. The large Δt imposed on the porous media indicated that the thermal physical properties of porous media no longer conformed to the perturbation theory.

Figures (6) and (7) represent the temporal relation of porous media (glass-water) Nusselt number (Nu_e) for three values of void space. They have been described same relationship and response for previous two figures. This is due to the Nusselt number was proportional and dependent to the value of onset of the heat transfer coefficient. The value of Nu_e for void space 2mm was in the range 0.61 with heat flux 400W/m^2 at time 295 seconds, and 1.8 with heat flux 800W/m^2 at time 180 seconds. It has been shown; these values were the lower values of Nu_e, which were calculated than others with void space of 3mm and 1mm at the same time of heating, and with the same values of heat flux. That was confirming with results of many authors such as (Tan and Sam, 1999). This is also indicated that the porous media (glass-water) with microstructure has void space of fluid layer 2mm, and in parallel configuration was a good insulation than others configuration design. These relations in two figures dependent upon the effective thermal properties of two phases (solid-fluid).

Figures (8) and (9) confirm the results of figures (6) and (7), which were indicated same relation and response of Nu_e, which was calculated through the macroscopic model of heat rate dependent upon the effective thermal properties. It has been pointed the relationship between the Nusselt number of fluid with time interval of heating. This was dependent upon the thermal conductivity of fluid phase (water). That value of Nu_f with Darcy number gave the value of Rayleigh number of porous media for every interval time of heating.

Figures (10) and (11) describe the relationship between Nusselt numbers (Nu_e) of porous media, which was dependent upon effective thermal conductivity of two phases and the Rayleigh number of porous media (glass-water) for three values of a void space of fluid layer with two values of heat flux. Results values of Rayleigh number of glass water was between 72.741 for 1mm void space, and 1946.563 for 3mm void space of fluid layer. That was consistent with results of many authors, which were presented by (Hans, 1999). These figures illustrate that the Ra_m values are seemed a constant through increasing the values of Nu_e for three values of void space with two values of heat flux. The variable of Ra_m values through time step was a very little pit. It has been shown higher value of Nu_e was indicated with 800W/m^2 than with heat flux of
400W/m². It has been noted a good response value of $R_a m$ for 2mm void space, and it has about 490 to 558. This value between higher value of $R_a m$ for 3mm, which was 1860 to 1950, and lower value of $R_a m$ for 1mm, which was 71 to 72 with two values of heat flux and at different time step of heating.

Figure (4) temporal variation of onset heat transfer coefficient in glass-water at different void space with heat flux 400W/m².

Figure (5) temporal variation of onset heat transfer coefficient in glass-water at different void space with heat flux 800W/m².

Figure (6) temporal variation of Nusselt number in glass-water at different void space with heat flux 400W/m².

Figure (7) temporal variation of Nusselt number in glass-water at different void space with heat flux 800W/m².
Figure (8) temporal variation of Nusselt number of fluid phase in glass-water at different void space with heat flux $400\text{W/m}^2$.

Figure (9) temporal variation of Nusselt number of fluid phase in glass-water at different void space with heat flux $800\text{W/m}^2$.

Figure (10) Nusselt number various Rayleigh number relationship found for glass-water at different void space with heat flux $400\text{W/m}^2$.

Figure (11) Nusselt number various Rayleigh number relationship found for glass-water at different void space with heat flux $800\text{W/m}^2$. 
4. Conclusion

1. The onset of transient convection in porous media has been successfully simulated by design mathematical macroscopic model, which was proposed for a semi-infinite saturated glass-water. This model dependent upon the macroscopic thermal effective properties in order to predict different parameters. These were pointed for the onset of natural convection inside the microstructure of porous media. This model give a better understanding of the realistic results, especially for presentation of Rayleigh number (Ra_m), and Nusselt number (Nu_e) for porous media. It has been used the explicit technique through the numerical solution of this model.

2. The Nusselt number of porous media (Nu_e) is defined as the ratio of heat flux by convection to the flux by convection. It has been shown from the prediction model; the acceptable value of (Nu_e) is in the average range value 8.5 to 3.6. This was simulated at assumption of the void space fluid layer was about 2mm and 3mm.

3. It has found from simulation that the glass-water porous media, which has a void space of water layer 2mm and in parallel configuration was a good insulation, can be used.

4. It is clear from the foregoing consideration that Rayleigh number of porous media (Ra_m) can only be accurately at very strict condition. Where the thermal diffusivity of the solid and liquid matrix are similar and the permeability of the porous media is large enough. So, that Δt or heat flux will be small.

5. Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>Diameter of beads</td>
<td>[m]</td>
</tr>
<tr>
<td>Da</td>
<td>Darcy number</td>
<td></td>
</tr>
<tr>
<td>g</td>
<td>Acceleration of gravity</td>
<td>[m/s²]</td>
</tr>
<tr>
<td>h</td>
<td>Convection heat transfer coefficient</td>
<td>[W/m²°C]</td>
</tr>
<tr>
<td>H</td>
<td>Layer thickness</td>
<td>[m]</td>
</tr>
<tr>
<td>i, j</td>
<td>Indices increase along x, y axis</td>
<td></td>
</tr>
<tr>
<td>k_e</td>
<td>Effective thermal conductivity of porous media</td>
<td>[W/m°C]</td>
</tr>
<tr>
<td>k_f</td>
<td>Thermal conductivity of fluid phase</td>
<td>[W/m°C]</td>
</tr>
<tr>
<td>k_s</td>
<td>Thermal conductivity of solid phase</td>
<td>[W/m°C]</td>
</tr>
<tr>
<td>K</td>
<td>Permeability</td>
<td>[m²]</td>
</tr>
<tr>
<td>Nu_e</td>
<td>Nusselt number of porous media</td>
<td></td>
</tr>
<tr>
<td>Nu_f</td>
<td>Nusselt number of fluid phase</td>
<td></td>
</tr>
<tr>
<td>Ra_f</td>
<td>Rayleigh number of fluid</td>
<td></td>
</tr>
<tr>
<td>Ra_m</td>
<td>Rayleigh number of porous media</td>
<td></td>
</tr>
<tr>
<td>t</td>
<td>Time</td>
<td>[second]</td>
</tr>
<tr>
<td>T</td>
<td>Temperature</td>
<td>[°C]</td>
</tr>
<tr>
<td>c_p</td>
<td>Specific heat at constant pressure</td>
<td>[J/kg.°C]</td>
</tr>
<tr>
<td>q</td>
<td>Heat flux</td>
<td>[W/m²]</td>
</tr>
<tr>
<td>α_e</td>
<td>Effective thermal diffusivity</td>
<td>[m²/s]</td>
</tr>
<tr>
<td>α_f</td>
<td>Thermal diffusivity of fluid</td>
<td>[m²/s]</td>
</tr>
<tr>
<td>ρ_f</td>
<td>Density of fluid phase</td>
<td>[kg/m³]</td>
</tr>
<tr>
<td>ρ_s</td>
<td>Density of solid phase</td>
<td>[kg/m³]</td>
</tr>
<tr>
<td>(ρ_c_p)_e</td>
<td>Effective heat capacity of porous media</td>
<td>[J/m³.°C]</td>
</tr>
</tbody>
</table>
Convergence factor. \( \lambda \)
Porosity. \( \varnothing \)
Volume coefficient of expansion. [1/K] \( \beta \)
Dynamic viscosity of fluid. [kg/m.s] \( \mu_f \)

6. References