Longitudinal electron scattering form factors for $^{50,52,54}$Cr

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Abstract.

The form factor for inelastic electron scattering to 2$^+$ and 4$^+$ states in $^{50,52,54}$Cr have been studied in the framework of shell model. The calculation is performed in the \((0f7/2,1p3/2,0f5/2,1p1/2)\). Longitudinal C2 and C4 multipolarity are investigated for these states. Core polarization effect are included through the first order perturbation theory and the matrix element are calculated with MSDI. The inclusion of core polarization leads to an enhancement of the calculated form factor, improving good agreement with experimental data.

1. Introduction

The calculations of shell model, carried out within a model space in which the nucleons are restricted to occupy a few orbits are unable to reproduce the measured static moments or transition strengths without scaling factors. Comparison between calculated and measured longitudinal electron scattering form factors has long been used as stringent tests of models for transition densities. Various microscopic and macroscopic theories have been used to study excitations in nuclei (Sato et al.,1985). Shell model within a restricted model space is one of the models, which succeeded in describing static properties of nuclei, when effective charges are used. Calculations of form factors using the model space wave function alone is inadequate for reproducing the data of electron scattering (Booten et al.,1994). Therefore, effects out of the model space, which are called core polarization effects, are necessary to be included in the calculations. The intermediate one-particle one-hole states are taken up to 6$\pi\rho$ excitation. These effects are found essential for obtaining a quantitative agreement with the experimental data (Yokoyama et al.,1989;Sato et al.,1994). A microscopic model (Radhi et al.,2001;Radhi,2003) has been used in order to study the core polarization effect on the longitudinal form factors of fp-shell nuclei. The author adopted the first order core polarization to calculate the C2 form factors of the fp-shell nuclei. Inelastic Electron Scattering from fp shell nuclei had been studied by Sahu et al (Saho et al.,1986). They calculated form factors for $^{50,52,54}$Cr, $^{54}$Fe, $^{56}$Fe, $^{46,48}$Ti, and $^{50}$Ti by the use of Hartree-Fock theory, their results are in a good agreement with the experimental data. The form factors for the inelastic electron scattering to 2$^+$, 4$^+$ and 6$^+$ states in $^{46,48,50}$Ti, $^{50,52,54}$Cr and $^{54,56}$Fe were studied by Sahu (Saho et al.,1990;Saho et al.,1987) in the framework of the Hartree-Fock model, also the calculation is performed in the 1f7/2, 2p3/2, 1f5/2, 2p1/2 model space using a modified Kuo-Brown effective interaction. Magnetic dipole excitation of N = 28 isotones $^{48}$Ca, $^{50}$Ti, $^{52}$Cr,
$^{54}\text{Fe}$ and $^{51}\text{V}$, was studied by Muto and Horie (Muto et al., 1985), in terms of the shell model by assuming $f^{m-n}(p_{3/2}f_{5/2}p_{1/2})^m$ configurations with $m = 0, 1$ and 2 on an inert $^{40}\text{Ca}$ core. The aim of present work is to use a realistic effective nucleon-nucleon (NN) interaction as a residual interaction to calculate the core polarization (CP) effects through a microscopic theory, with a selection of model space effective interaction which generates the model space wave functions (shell model wave functions) and highly excited states. The (MSDI) were used in this case as a residual interaction. The strength of the MSDI denoted by $A_f, B$ and $C$ are set equal to $A_0=A_1=B=6.2$ MeV and $C=0$. The single particle wave function were those of the harmonic oscillator potential (HO) with size parameter $b$ chosen to reproduce the measured ground state root mean square charge radii of these nuclei. The one-body density matrix (OBDM) elements ($\chi^2\Gamma_f\Gamma_i(\alpha_f,\alpha_i)$) are calculated using the shell model code OXBASH (Brown et al., 2005)

2. Theory

The electron scattering form factor for a given multipolarity $\lambda$ and momentum transfer $q$ is expressed as (de Forest, 1966)

$$|F_{\lambda}(q)|^2 = \frac{1}{2J_f+1}\left(\frac{4\pi}{Z^2}\right)^2|\Gamma_f\Gamma_i\hat{\chi}_{\lambda}\Gamma_i\hat{\chi}_{\lambda}|^2 F_{f,s}F_{c,m}^2 \ldots \ldots \ldots (1)$$

Where $F_{f,s} = e^{-0.43q^2/4}$ is the finite nucleon-size correction and $F_{c,m} = q^2b^2/4A$ is the center of mass correction, $A$ is the mass number and $b$ is the harmonic oscillator size parameter.

The effect of the core polarization on the form factors is based on a microscopic theory, which combines shell-model wave functions and configuration with higher energy as particle-hole perturbation expansion. The reduced matrix element of the electron scattering operator $\hat{T}_{\lambda}$ is expressed as a sum of the fp-model space (fp) contribution and the core-polarization (cp) contribution, as follows (Radhi et al., 2001)

$$\langle \Gamma_f \hat{T}_{\lambda} \Gamma_i \rangle = \langle \Gamma_f \hat{T}_{\lambda} \Gamma_i \rangle_{ms} + \langle \Gamma_f \hat{T}_{\lambda} \Gamma_i \rangle_{cp} \ldots \ldots \ldots (2)$$

with $\xi$ selection the longitudinal (L), electric (E) and magnetic (M) transverse form factors, respectively. The Greek symbols were used to denote quantum numbers in coordinate space and isospace, i.e. $\Gamma_i \equiv J_i T_i$, $\Gamma_f \equiv J_f T_f$ and $\lambda \equiv JT$. The fp-shell
model space element can be expressed as linear combination of the single-particle matrix element (Wildenthal, 1989)

\[ \langle \Gamma_f \parallel \hat{T}_\lambda^\varepsilon \parallel \Gamma_i \rangle_{ms} = \sum_{\alpha_j, \alpha_i} \chi^\varepsilon \Gamma_f \Gamma_i (\alpha_f, \alpha_f) \langle \alpha_f \parallel \hat{T}_\lambda^\varepsilon \parallel \alpha_i \rangle \]  

.....(3)

where \( \chi^\varepsilon \Gamma_f \Gamma_i (\alpha_f, \alpha_i) \) are the structure factors (one body density matrix element), given by,

\[ \chi^\varepsilon \Gamma_f \Gamma_i (\alpha_f, \alpha_i) = \frac{\langle \Gamma_f \parallel [\bar{a}(\alpha_f) \otimes \bar{a}(\alpha_i)]^\varepsilon \parallel \Gamma_i \rangle}{\sqrt{2\lambda + 1}} \]  

.....(4)

The \( \alpha_f \) and \( \alpha_i \) label are single-particle states for the fp-shell model space. Similarly, core-polarization matrix element as follow:

\[ \langle \Gamma_f \parallel \hat{\delta}T_\lambda^\varepsilon \parallel \Gamma_i \rangle_{cp} = \sum_{\alpha_j, \alpha_i} \chi^\varepsilon \Gamma_f \Gamma_i (\alpha_f, \alpha_i) \langle \alpha_f \parallel \hat{\delta}T_\lambda^\varepsilon \parallel \alpha_i \rangle \]  

.....(5)

Up to the first order perturbation theory, the single-particle matrix element for the higher-energy configuration is given by (Ellis et al., 1971)

\[ \langle \alpha_f \parallel \hat{T}_\lambda^\varepsilon \parallel \alpha_i \rangle = \langle \alpha_f \parallel \hat{T}_\lambda^\varepsilon \frac{O}{E_f - H_o} V_{res} \parallel \alpha_i \rangle + \langle \alpha_f \parallel V_{res} \frac{O}{E_i - H_o} \hat{T}_\lambda^\varepsilon \parallel \alpha_i \rangle \]  

.....(6)

The operator \( Q \) is the projection operator on the space outside the model space. \( E_i \) and \( E_f \) are the energies of initial and final states. For the residual interaction \( V_{res} \), the MSDI were adopted.

The two term in right hand side of Eq.(6) can be written as (Brussard, 1977)

\[ \sum_{\alpha, \alpha', \Gamma} \frac{(-1)^{\alpha_+ \alpha_+ \alpha_+ \Gamma}}{e_{\alpha_+} - e_{\alpha_+} - e_{\alpha_+} + e_{\alpha_+}} (2\Gamma + 1) \left\{ \frac{\alpha_f, \alpha_1, \lambda, \Gamma}{\alpha_2, \alpha_1, \Gamma} \sqrt{(1 + \delta_{\alpha_+, \alpha_+})(1 + \delta_{\alpha_+, \alpha_+})} \right\} \langle \alpha_f \parallel \hat{T}_\lambda^\varepsilon \parallel \alpha_i \rangle \]

\[ \times \langle \alpha_f \parallel \alpha_{\Gamma} \parallel V_{res} \parallel \alpha_{\alpha_2 \Gamma} \rangle \]

\[ + \text{Terms with \( \alpha_f \) and \( \alpha_i \) exchanged with an over all minus sign,} \]

.....(7)
Where $\alpha_1$ runs over particle states and $\alpha_2$ over hole state and $e$ is the single-particle energy. The hole states cover all $1s$ core orbits.

$$e_{nlj} = (2n+1 - \frac{1}{2})\eta\omega + \left\{ \begin{array}{l} \frac{1}{2}(l+1)\langle f(r) \rangle_{nl} \quad \text{for} \quad j = l - \frac{1}{2}, \\ \frac{1}{2}l\langle f(r) \rangle_{nl} \quad \text{for} \quad j = l + \frac{1}{2}, \end{array} \right.$$

$$\ldots \ldots (8)$$

With $\langle f(r) \rangle_{nl} \approx -20A^{-2/3}$ and $\eta\omega = 45A^{-1/3} - 25A^{-2/3}$

The electric transition strength is given by,

$$B(C\lambda) = \frac{|(2\lambda + 1)!|^2}{4\pi} \frac{Z^2}{k^2} |F_\lambda(k)|^2$$

Where $k = E_\lambda/\hbar c$.

3. Results and discussion

3.1. The nucleus $^{50}\text{Cr}$

The structure and properties of $^{50}\text{Cr}$ are experimentally and theoretically well studied. For the conventional multiparticle shell-model, $^{50}\text{Cr}$ has ten nucleons outside the core $^{40}\text{Ca}$ and it is possible to perform shell-model calculations for this nucleus in 2p1f shell space. The calculation in the 2p1f space assumes an inert $^{40}\text{Ca}$ core.

The two transitions under investigation are C2, 0.78MeV ($0^+ 1 \rightarrow 2^+ 1$) and C4, 2.675MeV ($0^+ 1 \rightarrow 4^+ 1$). The calculated $B(C2)$ and $B(C4)$ from the present work in comparison with experimental values are displayed in table (1).

3.1.1 The 0.78 MeV ($2^+ 1$) state

In this transition, the electron excites the nucleus from the ground state ($0^+ 1$) to the state ($2^+ 1$) with excitation energy of 0.78 MeV. In fig.(1) the experimental data of the C2 Coulomb form factors which are taken from Ref. (Sahal et al.,1990) are compared with the theoretical pf-shell model calculation. The solid curve shows the result with core-polarization effects and the dashed curve corresponding to the result without core polarization effects (pf-shell model space only). We are observed three peaks in the form factors in this nucleus. In the present of core polarization effect, the first peak is in good agreement with experimental data, but the second peak is overestimated and the third peak is underestimated. In pf-shell model, the calculated form factors underpredict the data in all regions of momentum transfers $q$, as shown in fig.(1) by dashed curve. In this model only model space wave function are considered. The pf-shell fail to describe
the data in form factors. Core polarization effects enhance the form factor and reproduce
the measured form factor up to $q=1.1$ fm$^{-1}$ as shown by solid curve of fig.(1).

Sahu et al. (Saho et al.,1990) have measured the form factors for the transition
$0^+ \rightarrow 2^+$ up to momentum transfers $q=3$ fm$^{-1}$. They also observed three peaks in the
form factor in this nucleus. They found that the experimental data are good agreement
with calculated form factor within Hatree-Fock model.

![Inelastic longitudinal form factors for the transition to the $2^+$ in the $^{50}$Cr. The experimental data are taken from ref. (Sahu et al.,1990)](image)

**Fig.(1) Inelastic longitudinal form factors for the transition to the $2^+$ in the $^{50}$Cr. The experimental data are taken from ref. (Sahu et al.,1990)***

### 3.1.2. The 2.675 MeV (4$^+$ 1) state

The form factor for C4 transition in $^{50}$Cr with an excited energy 2.675 MeV is
displayed in fig.(2), where the solid curve represent the model space with the effect of
core polarization, the dashed curve represent the model space and the experimental data
represented by points. The data are well reproduced for first lobe, and also up to
$q=1.4$ fm$^{-1}$. For $1.4<q<2$ fm$^{-1}$ the calculated form factors is overestimated and shifted
toward higher q values. As shown in fig.(2) the second maximum in underestimated.
When the core polarization effect is included we get a reasonable agreement between
calculated data and experiment for the first maximum, but fail to describe the form
factors for the second maximum. The model space fail to describe the data in the form
factors for all momentum transfers. Raina et al. (Raina et al.,1988) were also unable to

(continued on next page)
reproduce the second maximum for $^{50}\text{Cr}$ within their projected Hartree-Fock-Bogoliubov formalism, where we get a similar result.

![Graph](image)

Fig.(2) Inelastic longitudinal form factors for the transition to the $4^+$ in the $^{56}\text{Cr}$, the experimental data are taken from ref. (Raina, 1988)

### 3.2 The nucleus $^{52}\text{Cr}$

For Chromium fifty two the core is considered as $^{48}\text{Ca}$ and four particles are distributed over 2p1f-shell model space. Two transitions are considered in this work, namely:

- C2, 1.43MeV ($0^+ \rightarrow 2^+$) and C4, 2.600MeV ($0^+ \rightarrow 4^+$). The calculated $B(C2)$ and $B(C4)$ from the present work in comparison with experimental values are displayed in table (1).
3.2.1. The 1.43 MeV (2\(^+\) 2) state

The nucleus is excited from the ground state (0\(^+\) 2) to the state (2\(^+\) 2) with an excitation energy 1.43 MeV. Fig.(3) shows the calculated longitudinal Coulomb C2 electron scattering form factor as a function of momentum transfers q. The dashed curve represents the results of 2p1f-shell, while the solid curve represents the results of 2p1f-shell with the inclusion of core polarization effects. For this nucleus, the form factor show three peaks. The first peak occurs at 0.7 fm\(^{-1}\), the second at q=1.7fm\(^{-1}\) and the third peak at q=2.6fm\(^{-1}\). The first peak is reasonably well reproduced within our calculation up to momentum transfers q=1 fm\(^{-1}\). The second peak is overestimated, but the third maximum is quenched. In general the behavior qualitatively is in a good agreement with the experiment. The 2p1f-shell model fail to describe the data in all momentum transfers and the inclusion of cp effects enhance the form factor. Sahu et al. (Sahu et al.,1990) also observe three peaks in the form factor in these nucleus, where they observe that the first two peaks are reasonably well reproduced within Hartree-Fock model, but the third is overestimated.

Fig.(3) Inelastic longitudinal form factors for the transition to the 2\(^+\) in the \(^{52}\text{Cr}\). The experimental data are taken from ref. (Sahu et al.,1990)

3.2.2. The 2.600 MeV (4\(^+\) 2) state

Fig.(4) compares the calculated and observed form factor for the 0\(^+\) \rightarrow 4\(^+\) transition of nucleus \(^{52}\text{Cr}\) with inclusion of cp effect as shown by solid curve and that
without cp effect as shown by dashed curve in fig.(4). The agreement between the experimental data and the result of 2p1f-shell model with the inclusion of cp effect in the region of q<1.5 fm$^{-1}$ is quite good both in behavior and magnitude. There may be some disagreement with the experimental data in the region of q>1.5fm$^{-1}$, where the position of the first minimum is shifted toward higher momentum transfers and the calculation underestimated the magnitude of form factors around its second maximum. The 2p1f-shell model space well reproduce the second maximum, the first maximum is underestimated as shown by dashed curve in fig.(4). Raina et al. observed similar behavior within the projected Hartree-Fock- Bogoliubov formalism (Raina et al., 1988).

3.3. The nucleus $^{54}$Cr

Chromium $^{54}$Cr has been extensively studied both theoretically and experimentally. For the conventional many particle shell model, this nucleus is considered as an inert $^{48}$Ca core plus six nucleons distributed over 2p1f space. The calculations are presented for following transitions C2, 0.84 MeV ($0^+\rightarrow 2^+$) and C4, 0.832 ($0^+\rightarrow 4^+$). The calculated $B(C2)$ and $B(C4)$ from the present work in comparison with experimental values are displayed in table (1).
3.3.1. The 0.84 MeV (2\(^+\)) state

The form factor for C2 transition in \(^{54}\)Cr with an excitation energy 0.84 MeV, when the electron excite the nucleus from ground state (0\(^+\)) to (2\(^+\)) state. A good results to the \(^{54}\)Cr data is obtained with the 2p1f-shell model calculations including cp effects in either first or third maxima, but the second maximum is overestimated as shown in fig.(5) by solid curve. In all region of momentum transfers the form factor is predicted very well in shape. Sahu et al (Sahu et al.,1990) have measured the form factors for the transition 0\(^+\) \(\rightarrow\) 2\(^+\) for \(^{54}\)Cr up to momentum transfers 3fm\(^{-1}\). They also observed three peaks in the form factors in this nucleus. They found that that the data could be available within Hartree-Fock model for the first and second maxima, but the third maximum is overestimated. In our calculation we get a good fit between calculated and experimental form factors for the third maximum. The model space fail to describe the form factors in all momentum transfers, so the present of cp effect enhance the form factor.

![Graph showing form factors for \(^{54}\)Cr transition](image)

Fig.(5) Inelastic longitudinal form factors for the transition to the 2\(^+\) in the \(^{54}\)Cr, the experimental data are taken from ref.(Sahu at al.,1990)

3.3.2. The 0.832 MeV (4\(^+\)) state

In this state the electron excite the nucleus from ground state (0\(^+\)) 3 to the excited state (4\(^+\)) 3. Two peaks we observe as shown in fig.(6) with and without cp effect. It is seen that the present calculation is quite successful in reproducing the
magnitude of the form factors at the first maxima with inclusion of cp as shown in fig.(6) by solid curve. However, one observe discrepancies in the momentum transfers range $1.8 < q < 2.6$ fm$^{-1}$, the calculation overestimate the magnitude of the form factors around its second maximum. Raina et al. (Raina et al., 1988) observe the position of the first minimum is shifted towards higher momentum transfers and the calculation underestimated the magnitude of form factor around its second maximum within their projected Hatree-Fock-Bogoliubov formalism. As shown in fig.(6) the 2p1f-shell model space calculation fail to describe the data in all the region of momentum transfers, so the inclusion of cp effect lead to an enhancement in the calculation of the form factors.

Fig.(6) Experimental and calculated form factor for the transition to the $4^+$ in the $^{54}$Cr. The experimental data are taken from ref.(Raina et al., 1988)
Table(1): Theoretical calculations of the reduced transition probabilities $B(C2)$ (in units $e^2 fm^4$) and $B(C4)$ (in units of $e^4 fm^6$) in comparison with experimental values.

<table>
<thead>
<tr>
<th>Exp.[Ref.]</th>
<th>$fp^{+} cp$</th>
<th>$fp$</th>
<th>$(E_x MeV)$</th>
<th>$T_f$</th>
<th>$J^+_f$</th>
<th>$T_i$</th>
<th>$J^+_i$</th>
<th>Nucleus</th>
</tr>
</thead>
<tbody>
<tr>
<td>1020 ± 30 9(Toweley,1975) 0.451E + 5(Sahoetal,1990)</td>
<td>1141 0.3839E + 5</td>
<td>16 0.5355E + 4</td>
<td>0.78 2.675</td>
<td>1 1</td>
<td>2$^+$</td>
<td>1</td>
<td>0$^+$</td>
<td>$^{50}$Cr</td>
</tr>
<tr>
<td>660 ± 30 (Toweley,1975) 0.101E + 6(Sahoetal,1990)</td>
<td>930 0.1343E + 6</td>
<td>12.81 0.4284E + 4</td>
<td>1.43 2.37</td>
<td>2 2</td>
<td>2$^+$</td>
<td>2</td>
<td>0$^+$</td>
<td>$^{52}$Cr</td>
</tr>
<tr>
<td>949 (Lightboody,1983) 0.167E + 6 (Sahoetal,1990)</td>
<td>1093 0.6265E + 5</td>
<td>0.13 6.861</td>
<td>0.84 1.82</td>
<td>3 3</td>
<td>2$^+$</td>
<td>3</td>
<td>0$^+$</td>
<td>$^{54}$Cr</td>
</tr>
</tbody>
</table>

4. Conclusions

The fp-shell models, which can describe static properties and energy level are less successful in describing dynamic properties such as C2 and C4 transition rates and electron scattering form factors. The inclusion of higher-excited configurations by means of cp enhances the form factors and brings the theoretical results closer to the experimental data.

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