Transient Behavior of Nanofluid Natural Convection in Equilateral Triangular Enclosure

Tahseen A. Al-Hattab  
College of Engineering  
Babylon University  
Alhattab.t@gmail.com

Jaleel M. AlGhuraby  
Karbalā Technical Institute  
Al-Furat Al-Awsat Technical University  
jaleel_99@yahoo.com

Hassanin M. Ali  
College of Engineering  
Babylon University  
Hassanin.chemical@gmail.com

Abstract:
In this work, the unsteady state behavior of natural convection of nanofluid in triangular enclosure was studied. Numerical solution of laminar convection of Cu-Water nanofluid over a range of Rayleigh numbers ($10^3$–$10^7$) and volume fraction of nanoparticles (0–20%) were performed. The Non-dimensional form of the energy and the momentum equations are solved using the commercial package COMSOL (5). A uniform wall temperature (UWT) boundary condition was subjected to the enclosure. The base temperature was always higher than the inclined surfaces. The effects of Rayleigh number on the velocities, temperature and Nusselt number were examined. The effects of volume fraction of nanoparticles on thermal performance and the stability of the transient behavior of the heat transfer and the fluid flow were examined. Based on the numerical results, it was shown that the addition of Cu nanoparticles enhances the heat transfer rate for all values of Rayleigh. Moreover, it was found that both the stability of the transient behavior of the natural convection and the steady state profiles of temperature and velocities were affected by the existence of the Cu nanoparticles.

Keywords: transient, natural convection, triangular enclosure, nanofluid, Cu, COMSOL.

I. Introduction
Due to its wide applications, the natural convection is one of the most important phenomena in thermal engineering systems. The natural convection heat transfer is a main design parameter in many applications [Yang, 1987; Raithby et al., 1998; Jaluria, 2003].

Different technique can be used to improve and enhance the heat transfer. Using the nanofluids is one of the promising methods. The conventional heat transfer fluids have low thermal conductivity that can be improved by using a dispersion of nanoparticle in the base fluid. The nanofluids have been studied extensively in many fields such as cooling of electronics [Turgut et al., 2014; Vasu et al., 2009], solar water heating [Mahian et al., 2013], nuclear reactor cooling [Nematolahi et al., 2015; Buongiorno et al., 2009; Buongiorno et al., 2008], chillers [Anandakumar, 2015; Liu et.

الخلاصة:
يتعلق هذا البحث إلى دراسة التصرف غير المستقر لانتقال الحرارة بالحمل الحر في مائع نانوي في حيز مغلق مثلث الشكل.

I. مقدمة
بفضل إعداد حفاضات، فإن الانتقال الحراري بالحمل الحر في حيز مغلق مثلث الشكل يتمثل بالعديد من الأمثلات، والتي تمثل حثًا لبحثنا على التأثيرات التي تكون عنصرًا في التأصيل العملية للذوبان، حيث يمكن استخدام برنامج (COMSOL 5) في حل المعادلات الخطية والجهادية المرتبطة بوجود تدفق نانوي في حيز مغلق مثلث الشكل. كما يمكن استخدام الناحية في حل مشكلة الحرق الذي يتم من خلال تسهيل التدفق الحراري للذوبان في حيز مغلق مثلث الشكل.

كلمات المفتاحية: غير مستقر، حامل طبيعي، حيز مغلق مثلث، مائع نانوي، نانوس، كومسول
Recently, natural convection was studied in an isosceles triangular enclosure with a heat source located at its bottom wall and filled with an Ethylene Glycol–Copper nanofluid [Ghasemi et al., 2010]. A numerical analysis of unsteady state natural convection heat transfer in an isosceles triangular cavity filled with Al₂O₃-water nanofluid was carried out with non-uniform hot temperature on the bottom wall of the cavity [Rahman et al., 2014].

In this work, an attempt was made to investigate the natural convection in a triangular enclosure of copper water nanofluid subjected to constant different temperature at the lower and the upper halves. The effect of addition of nanoparticles on the stability of thermal behavior of the enclosure will be examined.

I. PROBLEM DESCRIPTION

Fig. (1) shows the geometry of the triangular enclosure filled with nanofluids. The inclined wall of the enclosure is maintained at a uniform low temperature (T₁) while the bottom wall is maintained at a uniform high temperature (T₂). The enclosure is filled with a water-Cu nanofluid with different volume fraction of solid particles. It is assumed that the flow is laminar and the fluid is Newtonian and incompressible. Also it is assumed that the base fluid and the nanoparticles are in thermal equilibrium. The thermophysical properties of the nanoparticles and base fluid are given in Table 1. Constant thermophysical properties are considered for the nanofluids except for the density variation in the buoyancy forces determined by using the Boussinesq approximation.

<table>
<thead>
<tr>
<th>Property</th>
<th>Cu</th>
<th>Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cp (J/kg K)</td>
<td>385</td>
<td>4179</td>
</tr>
<tr>
<td>ρ (kg/m³)</td>
<td>8933</td>
<td>997</td>
</tr>
<tr>
<td>k (W/m K)</td>
<td>400</td>
<td>0.61</td>
</tr>
<tr>
<td>β (1/K)</td>
<td>1.67x10⁻³</td>
<td>2.1x10⁻⁴</td>
</tr>
</tbody>
</table>

The thermal conductivity and the viscosity of the nanofluid are given by the following models:

\[
\frac{k_{\text{nfl}}}{k_f} = \frac{k_p + 2k_f - 2(k_f - k_p)\varphi}{k_p + 2k_f + (k_f - k_p)\varphi} \quad (1)
\]

\[
\mu_{\text{nfl}} = \frac{\mu_f}{(1-\varphi)^{2.5}} \quad (2)
\]
The governing equations for a Boussinesq incompressible laminar flow under unsteady state conditions in a two-dimensional geometry take the following form:

**Continuity equations:**
\[ \nabla \cdot \mathbf{u} = 0 \]  

**Momentum equations:**
\[ \frac{\partial u}{\partial t} + (u \cdot \nabla) u = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \nabla^2 u + \nabla \left[ \frac{1}{2} (\mathbf{u} \cdot \mathbf{u}) \right] + \mathbf{F} \]

Where \( \mathbf{F} = \left\{ \begin{array}{c} 0 \\ \mathbf{g} \theta \left( \frac{T - T_c}{T_h - T_c} \right) \end{array} \right\} \)

**Energy equation:**
\[ \frac{\partial T}{\partial t} + (u \cdot \nabla) T = \nabla \cdot (\alpha_r \nabla T) \]

The following dimensionless variables are used:

\[
\begin{align*}
X &= \frac{x}{L}; & Y &= \frac{y}{L}; & \tau &= \frac{t \alpha_f}{\nu L^2} \\
U &= \frac{u L}{\alpha_f}; & V &= \frac{v L}{\alpha_f}; & P &= \frac{\rho L^2}{\rho_f \alpha_f^2}; & \theta &= \frac{T - T_c}{T_h - T_c} \\
k &= \frac{k_n}{k_f}; & \alpha &= \frac{\alpha_n}{\alpha_f}; & \mu &= \frac{\mu_n}{\mu_f};
\end{align*}
\]

The governing equations are re-written in above dimensionless form as follows:

\[ \nabla \cdot \mathbf{U} = 0 \]  

\[ \frac{\partial U}{\partial t} + (U \cdot \nabla) U = (\chi(\rho))^{-1} \nabla \cdot \left[ -p I + \mu_f \nabla U + (\nabla U)^T \right] - \frac{1}{2} \mu F \nabla (\mathbf{U} \cdot \mathbf{U}) + \mathbf{F}' \]

\[ \frac{\partial T}{\partial t} + (U \cdot \nabla) T = \nabla \cdot (\alpha_r \nabla T) \]

where \( \mathbf{F}' = \left\{ \begin{array}{c} 0 \\ (R_a \text{Pr} \chi(\beta) \theta) \end{array} \right\} \)

and \( \chi(\lambda) = 1 - \varphi + \varphi \chi \frac{\lambda}{h} \) is the property function; e.g;

\( \chi(\beta) = 1 - \varphi + \varphi \frac{\theta}{h} \)

The following dimensionless initial boundary conditions are imposed:

**Initial conditions**
\[ \mathbf{u}(0) = \mathbf{0}; \quad \theta = \theta_0; \quad U = 0; \quad V = 0 \]  

**Boundary conditions**
\[ \theta = 0; \quad U = 0; \quad V = 0 \quad \text{at} \ \Gamma_1 \]
\[ \theta = 1; \quad U = 0; \quad V = 0 \quad \text{at} \ \Gamma_2 \]

The local Nusselt number along the hot wall is defined as

\[ \mathrm{Nu} = \frac{1}{L} \left. \frac{\partial T}{\partial x} \right|_{\Gamma_2} = -k \left. \frac{\partial \theta}{\partial x} \right|_{\Gamma_2} \]  

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and the average Nusselt number is expressed as

\[ \text{Nu}_{av} = \int_0^L \text{Nu} \, dX \quad (12) \]

Fig. (1): Physical model.

II. NUMERICAL SOLUTION

With the aid of COMSOL Script, a code was written to solve the set of the dimensionless governing equations (6-8) and the associated initial and boundary conditions. The program COMSOL uses the finite element method to solve multiphysics problems. In order to have an independent grid size numerical solutions, The numerical solution was carried out by applying an optimum number of triangular element in order to ensure a grid independent solution. Fig.(2) shows the effect of grid size represented as a (Number of Elements) on the value of the Nusselt number. It is clearly shown that is suitable to have convenient solutions of this problem with a grid of about \(10^3\) elements.

III. RESULTS AND DISCUSSION

Based on the numerical simulations, the transient behavior of the heat and fluid flow of nanofluid is investigated in the equilateral triangular enclosure. A uniform heating with constant temperature through the horizontal and uniform cooling through the inclined surfaces is prescribed. The effect of Rayleigh number and nanoparticle volume fraction are discovered.

A typical example of this problem for Rayleigh number of \(1 \times 10^6\) for clear water with \(Pr=6.2\) is illustrated in Fig. (3). The isotherms and the stream lines are shown at different time steps. It is shown that at the first time steps there were three small plumes of temperature isotherms appeared and then collapsed to a large stable plume.

Fig. (4) shows the steady state solution of problem for different values of for Rayleigh number equal and less than \(1 \times 10^6\). At \(Ra=10^3\) the conduction is dominated as a mode in heat transfer. As \(Ra\) exceeds a value of \(10^3\) the heat transfer controlled by natural convection. The temperature isotherms and velocity streamlines confirm such behavior. It is observed that this solution is symmetric with respect to the geometric midline of the enclosure.

The solution image is characterized by single plume in temp isotherm and two cells rotating in opposite directions for velocity streamlines.

As the Rayleigh number exceeds a value of \(10^6\), Fig. (5), the thermal behavior is observed to be unstable with unsustainable periodic variation in temperature-velocities profiles.
The numerical analysis was extended to investigate the heat transfer coefficient on the surfaces of the enclosure. Figure (6) shows the average Nusselt numbers on the base of the enclosure with time for different values of $Ra$. The calculated results exhibit similar features as previously discussed.

The effect of addition of nanoparticles to the base fluid of water on the average Nusselt numbers can be explained with the aid of Fig. (7). It can be shown that the existence of nanoparticles will not only enhance the heat transfer but also stabilize the thermal behavior. As ($\chi$) increases, the system becomes stable and the Nu increase also. At $Ra=4 \times 10^6$, the average value of the average Nu is about (24) for pure water whereas the enhancement due to addition of copper nanoparticles with (20%) volume fraction reaches to 1.5 times than they are in the pure water at the same $Ra$.

Fig.(2): Transient Average Nusselt number at different values of Number of Elements

Fig.(3): Snapshot of dynamic behavior of clear water for $Ra=1 \times 10^6$. 

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Fig.(4): Steady state profile of clear water at different values of Ra.

Fig.(5): Snapshot of dynamic behavior of clear water for $Ra=7 \times 10^6$. 

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IV. CONCLUSIONS

The numerical analysis of the transient behavior of natural convection heat transfer of copper-water nanofluid filled an equilateral triangular cavity has been done in this work. The convection fluid flow through the enclosure was caused by the potential difference in temperature between the upper (cold) and the lower (hot) sides of the cavity. The commercial package COMSOL Multiphysics (5.0) was used to formulate and solve the model equations of the problem. The computations in this work include the effects of some factors such as the volume fraction ($\chi$) of solid particles of nanoparticles (Cu) and Rayleigh number (Ra), based on base fluid (water), on the temperature, velocities and the Nusselt number (Nu). It's observed that the existence of nanoparticles make the transient behavior of the natural convection (velocities and temperature) for nanofluid more stable than the behavior for the pure fluid. Moreover it is found that for all values of solid volume fraction of nanoparticles, increasing Rayleigh number results in enhancing the heat transfer and the Nusselt number due to strengthening of the bouncy forces.

![Graph](image)

**Fig.(6):** Variation of average Nusselt Number with time for different values of Rayleigh number

![Graph](image)

**Fig.(7):** Variation of average Nusselt Number with time for different values of volume fraction ($\phi$) at Rayleigh number (a) $Ra=3\times10^6$ (b) $Ra=4\times10^6$
NOMENCLATURE

<table>
<thead>
<tr>
<th>Cp</th>
<th>Specific heat (J /kg K)</th>
<th>T</th>
<th>Temperature (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>g</td>
<td>Gravity acceleration (m/s²)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gr</td>
<td>Grashof number (g β(T_h-T_c) L³/ν²)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>k</td>
<td>Thermal conductivity (W /m K )</td>
<td>U</td>
<td>Dimensionless x component of velocity</td>
</tr>
<tr>
<td>L</td>
<td>Side length of the enclosure (m)</td>
<td>v</td>
<td>Velocity in y direction (m/s)</td>
</tr>
<tr>
<td>Nu</td>
<td>Nusselt number</td>
<td></td>
<td></td>
</tr>
<tr>
<td>p</td>
<td>Pressure (Pa)</td>
<td>x</td>
<td>Distance in x direction (m)</td>
</tr>
<tr>
<td>P</td>
<td>Dimensionless pressure</td>
<td>X</td>
<td>Dimensionless x coordinate</td>
</tr>
<tr>
<td>Pr</td>
<td>Prandtl number (μ Cp/k)</td>
<td>y</td>
<td>Distance in y direction (m)</td>
</tr>
<tr>
<td>Ra</td>
<td>Rayleigh number (Gr Pr)</td>
<td>Y</td>
<td>Dimensionless y coordinate</td>
</tr>
</tbody>
</table>

Greek symbols

| α | Thermal diffusivity (m²/s) |
| β | Thermal expansion coefficient (1/K) |
| T | Boundary of cavity |
| θ | Nondimensional temperature |
| μ | Dynamic viscosity (kg/ms) |
| ρ | Density (kg/m³) |
| τ | Nondimensional time |
| χ | Nanoparticle fraction |
| ν | Kinematic viscosity (m²/s) |

Subscripts

| c | cold |
| f | fluid |
| h | hot |
| n | nanofluid |
| p | particle |
| ∞ | environment |

REFERENCES


