Experimental Study for Hydraulic Characteristics of Flow over Compound Regular Notches

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Abstract

The hydraulic relationships of flow through notches of different common shapes were investigated in this study. The hydraulic performance of notches were carried out experimentally, by using eight models, the effect of various shapes of notches to specify a range of hydraulic parameters such as coefficient of discharge $C_d$, energy dissipation ratio $E\%$, and self-aeration of notch $AE\%$. The experimental results of this study showed that the dissipation energy corresponding to $V$-notches is larger than resulted from rectangular notches. For compound notches of rectangular and triangle, a turbulence in flow increases as the angle of lower notch decreases. Thus, this type with ($\theta=120^\circ$) is the most efficient between the studied models, which gives dissipation of flow energy ratio of (35.86%) for strong hydraulic jump, and (20.7%) for steady jump type. In addition, this type improved the self-aeration of flow given by ($\frac{Q_a}{Q}$%), a percent of 35.53 is measured. This work also reveal that length of roller and the length of hydraulic jump were directly proportional to the angle of compound notch. Depending on statistical basis, this study derived set of empirical relationships to estimate coefficient of discharge, energy dissipation ratio, and reduction of hydraulic jump length ratio with acceptable values of coefficient of determination.

Key Words: Compound notches, Coefficient of discharge, Energy dissipation of flow, Aeration efficiency, Hydraulic jump length.

1-Introduction

Dissipation of kinematic energy of flow through and over the hydraulic structures is essential to prevent erosion of downstream channel bed and banks, collapse of downstream structures. Hydraulic jumps used for dissipating this kinetic energy in a stilling basin through generation of a jump (Habib, 2013), generating large-scale turbulence, surface waves and spray. Causing more energy dissipation and makes, air
mixed into water. In water treatment, process hydraulic jump was formed in the location where self-aeration of flow is necessary to increase the content of dissolved oxygen of water, and supports in mixing of chemicals in these processes (Anandraj, 2012). Hydraulic structures such as weirs have aeration effect on the water body in downstream of the structures. Several studies on the aeration at free weirs are conducted and relation between aeration efficiency and local hydraulic properties has been cleared.

The type of hydraulic jump drawn as classical jump that occurs in wide-ranging rectangular horizontal channels with smooth boundaries has been extensively studied. Abdel-Aal et. al., (1998) studied the hydraulic jump within a diverging rectangular channel, it was found that the relative depth of free radial jump and the length of the jump were shorter than those formed in rectangular channels, while the rate of energy loss increases through the jump in radial basin compared to that in rectangular one. Hayawi and Al-Talib (2008) performed an experimental work to test efficiency of dissipating flow energy for stepped and un-stepped weirs. Results shows that increasing height of weir and decreasing both number of steps and downstream slopes stepped face of the weir will cause an increase of the ratio of flow energy dissipation, and the stepped weirs are more efficient in flow energy dissipation compared with un-stepped weirs. The percentage of flow energy dissipation was increased by increasing the ratio of height of weir to critical water depth, the ratio of length of the step to critical water depth and the ratio of height of step to critical water depth while it decreases by increasing the discharge. Hayawi et. al., (2009) studied experimentally the coefficient of discharge for a compound notches as hydraulic measuring device, it was found that the coefficient of discharge \( C_d \) increases as the ratio(head through the rectangular weir/diameter of the gate) \( h/d \) increase and for constant value of \( h/d \), the coefficient of discharge \( C_d \) increase as the width of rectangular weir increase, the values of \( C_d \) range from around 0.522 to 0.853 with an average of 0.695. Noori and Jaafar (2011) studied different crest shapes of rectangular side weirs on their hydraulic performance. Results shows that the average difference of energy between two ends of the side weir is very small (less than 1%) which neglected. Linear variation for coefficient of discharge and Froude number in upstream side, coefficient of discharge and ratio of upstream water depth to weir height, and discharge coefficient with the ratio of upstream head above crest-to-crest radius were detected. Lee et al., (2012) was experimentally studied a compound of regular notches sharp-crested weir, the combination of a trapezoidal weir, sloping crests and a rectangular weir. The discharges and velocity distributions over the weir were measured varying the upstream head. it was found that the kinetic energy correction factors decreased with increasing the head of the weir. The weir behaves as uniform flows passing over the weir under the conditions of the (flow depth/height of the trapezoidal weir with sidewall angle \( \theta_1 \geq 1.5 \)).

Arvanaghi and Oskuei (2013) performed experimentally a study of rectangular sharp crested weirs, and the influence of high Froude number and Reynolds number on discharge coefficient. In order to conduct a numerically parametric study with constant coefficient of discharge, the CFD commercial software Fluent v. 6.2 was used. the Fluent output devoted accurate modelling of flow through this weir types, for a specific value of three dimensionless parameters of weir height, Froude number and Reynolds number, discharge coefficient is tending to the fixed value of 0.7.

Habib (2013) introduced laboratory approach to investigate influence of the flow coming from the opening of the weir and the effect of the relative diameter of the opening
which used to convey much water to the downstream side on the hydraulic jump formed in a rectangular basin downstream the weir, also to develop equations to obtain the jump characteristics for the adopted case of the study.

Weir aeration occurs in rivers and water treatment processes. Frequently, the hydraulic head is available normally and attains null operating cost. In certain instances, however, self-aeration of weir is economically competitive with alternative technology as surface self-aeration, despite the costs of pumping energy were counted. The water flow through a weir or over waterfall structures was classified as a free jet just before breaking up into drops. Normally, almost the oxygen transfer is carried out in such type of structure throughout the breakdown of the jet, and the channel bed subjected to impact due to subsequent free jet. Moreover, due to increases in the hydrostatic pressure on the entrainment air bubbles the depth of the downstream water pool could be increased the absorption (Baylar, 2000).

Emiroglu (2006) reviewed and compared studies for self-aeration efficiency on the smooth and stepped chutes, in order to decide highly effective aerators in streams, rivers, constructed channels, and water treatment plants. Results showed based on the strong turbulent mixing associated with substantial air bubble entrainment, the stepped chutes are very efficient at oxygen transfer, especially in napped flow, which becomes more evident. Anandraj (2012) investigated hydraulic jump aeration efficiency beyond a sluice gate in laboratory tilting flume for various range of channel slopes and flow rates. Results indicated a linear correlation between the hydraulic jump aeration efficiency and the rate of energy dissipation per unit width of the channel. Present work looks at the hydraulic performance of notches, tests are carried out experimentally, and in particular, the effect of weir compound of regular notches to identify hydraulic characteristics, which are summarized as:
1- Discharge coefficient of proposed models.
2- Efficiency of dissipating flow energy.
3- Relative length of the hydraulic jump.
4- Self-aeration of notches.

2-Materials and Methods
Tests have been conducted for this research in the laboratory of hydraulic division in department of civil engineering, college of engineering, University of Babylon. The experimental flume as shown in figure(1) used in this study was 10m long with an adjustable slope, 0.3m width, and 0.45m depth. The channel consisted of toughened glass walls of 10mm thick, and a stainless steel floor. Two movable carriages with point gauges with accuracy of (0.5mm) were mounted on brass rails at the top of channel sides to measure the heads over weir and downstream the hydraulic jump. Water was supplied from a sump tank by a centrifugal pump with a maximum capacity 40 l/sec., raising water by pump from the storage tank to the flume then returns to ground tank by vertical outlet located in tail of flume. A flow meter was used to measure volume of flow. Flume bed was maintained at a horizontal slope during the tests.
Eight notches models were used to carry out the experimental work, these models were manufactured from a 10mm thick plywood sheets of the dimensions(40cm×30cm), which includes a standard rectangular notch and triangle notch with three different angles of (45°, 60°, 90°), in addition to compound notch models as shown in figure (2). All models dimensions and their designations are shown in Table (1) below.

3- Theoretical Analysis
3-1 Coefficient of Discharge (Cd):

The equation governing flow through weirs is given by (Arvanaghi, 2013):

\[ Q = kH^n \]  \hspace{1cm} (1)

In which;

- \( Q \): is the flow rate (L³/T).
- \( k \): is a parameter related to weir geometric shape and size.
- \( n \): weir shape number in which (n= 1.5 for rectangle weir notch; and n= 2.5 for triangular weir notch).
Table 1: Main characteristics of standard and compound notches

<table>
<thead>
<tr>
<th>Weir Model</th>
<th>Model No.</th>
<th>Dimensions (cm)</th>
<th></th>
<th></th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>Height</td>
<td>Width</td>
<td>Weir angle</td>
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<td>Rectangular</td>
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<td>25</td>
<td>20.0</td>
<td>--</td>
<td></td>
</tr>
<tr>
<td>Triangle</td>
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<td>25</td>
<td>20.7</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>25</td>
<td>28.8</td>
<td>60°</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>25</td>
<td>30.0</td>
<td>90°</td>
<td></td>
</tr>
<tr>
<td>Compound of regular notches</td>
<td>1</td>
<td>25</td>
<td>20.0</td>
<td>120°</td>
<td></td>
</tr>
<tr>
<td>Rectangle + Triangle</td>
<td>2</td>
<td>25</td>
<td>20.0</td>
<td>130°</td>
<td></td>
</tr>
<tr>
<td>Compound of regular notches</td>
<td>1</td>
<td>25</td>
<td>20.0</td>
<td>60°</td>
<td></td>
</tr>
<tr>
<td>Rectangle + Trapezoidal</td>
<td>2</td>
<td>25</td>
<td>20.7</td>
<td>90°</td>
<td></td>
</tr>
</tbody>
</table>

According to (Henderson, 1966), for rectangular sharp-crested weir, the head-discharge equation is:

\[ Q = \frac{2}{3} Cd \sqrt{2g WH^{3/2}} \]  

In which:
- \( W \): is the crest length of the weir (L).
- \( Cd \): is the coefficient of discharge.
- \( g \): the gravity acceleration (L/T²).
- \( H \): is the effective water head above weir crest (L).

According to (Piratheepan et al., 2006), the basic equation for a triangular sharp-crested weir expressed as:

\[ Q = \frac{8}{15} Cd \sqrt{2g \tan \frac{\theta}{2} H^{5/2}} \]  

In which:
- \( \theta \): is the notch angle.

For trapezoidal weirs, the flow equation is as follows (Lee et al., 2012):

\[ Q = \frac{2}{3} Cd \sqrt{2g H^{3/2} (L_1 + \frac{4}{5} H \tan \theta)} \]  

Where:
- \( Cd \): is the discharge coefficient.
- \( H \): is the head (L).
- \( L_1 \): is the bottom width of the cross section (L).
- \( \theta \): is the side slope angle.

Because a large discharge requires more depth as compared to other weirs and flumes for the same discharge. Therefore, measurement errors associated with the depth measurement have a significant effect on the discharge estimation. A compound weir and circular weirs both measure small discharges accurately while also having the capacity to measure the large discharge without large head requirements (Erickson et al., 2013).

Jan et al., (2006) experimentally studied four compound sharp-crested weirs, and confirmed that the linear combination method for developing the discharge equation of compound sharp-crested weirs is reasonable (quoted from Lee et al., 2012). A schematic diagram of the proposed compound rectangle and triangle notch was illustrated in figure (3). The cross section of the proposed compound weir consists of two areas, including areas \( A_{\text{rect.}} \), \( A_{\text{tri.}} \), theoretical discharge equation for the compound of regular
notches weir was derived by linear combination of basic head-discharge equations for triangular and rectangular notches.

\[
Q_1 = \sqrt{2g} \left[ \frac{8}{15} \tan \theta \left( Y_w \right)^{3/2} + \frac{2}{3} L_r (H - Y_w)^{3/2} \right]
\]

Figure (3): Definition sketch for notch of rectangle and triangle geometries

When the head was above the lower part of the weir \((H > Y_w)\), the discharge equation of compound notch of rectangle and triangle can obtained as follows:

\[
Q_1 = \sqrt{2g} \left[ \frac{8}{15} \tan \theta \left( Y_w \right)^{3/2} + \frac{2}{3} L_r (H - Y_w)^{3/2} \right]
\]

In which;

\(Q_1\): Total discharge for compound of regular notches weir of rectangle and triangle notches \((L^3/T)\).

Equation (5) can combined into a single global discharge coefficient \((Cd_1)\), and the resulting actual discharge relation is:

\[
Q_i = \sqrt{2g} Cd_i \left[ \frac{8}{15} \tan \theta \left( Y_w \right)^{3/2} + \frac{2}{3} L_r (H - Y_w)^{3/2} \right]
\]

In which;

\(Q_i\): Total actual discharge for compound of regular notches weir of rectangle and triangle notches \((L^3/T)\).

\(Cd_i\): discharge coefficient for compound of regular notches weir of rectangle and triangle notches. General definition sketch for the proposed compound of regular notches weir of rectangle and trapezoidal notches was showed in figure (4). The cross section of weir consists of two areas, including areas \(A_{\text{rect.}}\), \(A_{\text{trap}}\).

Figure (4): Definition sketch for compound notches of rectangle and trapezoidal geometries
When the head was above the lower part of the weir (H > Y_w), theoretical discharge equation obtained by the discharge relations of one trapezoidal weir with side slope angle (θ) and a rectangular weir as follows:

$$Q_2 = \sqrt{2g} \left[ \frac{2}{3} L_r (H - Y_w)^{3/2} + \frac{2}{3} Y_w^{3/2} \left( L_1 + \frac{4}{5} Y_w \tan \theta \right) \right]$$  ----- (7)

In which:
- Q_2: Total discharge for compound of regular notches weir of rectangle and trapezoidal notches (L^3/T).

Equation (7) can combined into a single global discharge coefficient (C_d), and the resulting actual discharge relation is:

$$Q = \sqrt{2g} C_d \left[ \frac{2}{3} L_r (H - Y_w)^{3/2} + \frac{2}{3} Y_w^{3/2} \left( L_1 + \frac{4}{5} Y_w \tan \theta \right) \right]$$  ----- (8)

Q: Total actual discharge for compound of regular notches weir of rectangle and trapezoidal notches (L^3/T).
C_d: discharge coefficient for compound of regular notches weir of rectangle and trapezoidal notches.

The discharge coefficient (C_d) taken as the ratio of actual discharge to the theoretical discharge, thus:

$$C_d = \frac{Q_{actual}}{Q_{theor.}}$$  ----- (9)

3-2 Energy dissipation efficiency:

Applying energy equations between two sections in upstream, and in downstream of weir, we get:

$$E_1 = H + \frac{V_1^2}{2g}$$  ----- (10)

$$E_2 = Y_2 + \frac{V_2^2}{2g}$$  ----- (11)

In which:
- E_1, E_2: are the flow energy at the upstream weir and at the downstream of hydraulic jump respectively (L).
- Y_2: water depth downstream hydraulic jump (L).
- V_1, V_2: flow velocity in upstream and downstream respectively (L/T).

Thus, the ratio of energy dissipation may expressed as follows:

$$E\% = \frac{(E_2 - E_1)}{E_1} \times 100$$  ----- (12)

3-3 Self-aeration efficiency:

Kucukali and Cokgor (2009) presented linear relationship between the efficiency of self-aeration and rate of energy dissipation as shown below [quoted from Anandraj, 2012]:

$$AE = 0.0015\psi + 0.01$$  ----- (13)

Where:
- $\psi = \frac{q \Delta H \gamma_w}{W}$  ----- (14)

$\psi$: is the rate of energy dissipation for unit width of channel (Watt/m).
\(q\): is the rate of flow of water for unit width of channel (L³/T/L).
\(\gamma_w\): is the specific weight of water (F/L³).
\(\Delta H\): is the losses in head through hydraulic jump (L).

\[
\Delta H = \left(H + \frac{V_1^2}{2g}\right) - \left(Y_2 + \frac{V_2^2}{2g}\right) \tag{15}
\]

4- Analysis and Discussion of Experimental Data

The analysis of experimental data intended to check the continuity of flow over the compound notches. Studying the flow characteristics such as the effect of primary Froude number \((F_{r1})\) and headwater ratio \((h_w/P_w)\) on the discharge coefficient for standard and compound of regular notches weir, the energy dissipation ratio (E%), self-aeration efficiency (AE%), and relative length of hydraulic jump \((L_j/Y_2)\), which can be detailed as:

4-1 \(C_d\) versus \((F_{r1}\ and\ \ h_w/P_w)\): The relations between discharge coefficient \((C_d)\) and Froude number \((F_{r1})\) is shown in figures (5 and 6). From these figures it can be seen that for each V-notch angle \((\theta)\), \((C_d)\) increases as \((F_{r1})\) increase. In addition, when V-notch is acute a maximum value of \((C_d)\) can be reached for acute angle of \((45°)\) and decrease as V-notch angle increase as for \((\theta=90°)\).

![Figure (5): Relation between discharge coefficients \((C_d)\) and Froude number \((F_{r1})\) for standard notches](image)

![Figure (6): Relation between discharge coefficients \((C_d)\) and Froude number \((F_{r1})\) for compound notches](image)
From figure (6) when compound rectangular and triangular notch is used, the lower V-notch is obtuse a maximum \( (C_d) \) can be reached for an obtuse angle \( (120^\circ) \). Rapid increase in coefficient of discharge as v-notch angle \( (\theta=130^\circ) \) for low values of primary Froude number \( (Fr_1<6.3) \). For compound notch of rectangular and trapezoidal, the maximum \( (C_d) \) can be reached at \( h_w/P_w=0.7 \) for V-notch angle of \((90^\circ)\), but \( (C_d) \) decreases for rectangular notch compared with both acute and obtuse V-notch.

![Figure (7): Relation between discharge coefficients \( (C_d) \) and \( (h_w/P_w) \) for standard notches](image)

From figure (8) when the parameter \( (h_w/P_w) \) increases, the \( (C_d) \) decreases for both angles of triangle notch of compound of regular notches, a minimum \( (C_d) \) can be recognized at more détente of V-notch angle. For the compound notches of rectangular and trapezoidal, the maximum \( (C_d) \) can be reached approximately at \( h_w/P_w=0.83 \) for a both angles of \((60^\circ, 90^\circ)\), beyond this value \( (C_d) \) decreases slightly as the parameter \( (h_w/P_w) \) increased.

![Figure (8): Relation between discharge coefficients \( (C_d) \) and \( (h_w/P_w) \) for compound notches](image)
4-2 Energy Dissipation Ratio (E\%) for Compound of regular notch Weirs:

Figures (9, and10) Shows the effect of notch angle (θ) and Froude number (Fr1) on energy dissipation ratio E\% for standard weirs. In figure (9), E\% approximately increases linearly when (Fr1) increases for the rectangular notch, while for the V-notch models (E\%) increases rapidly with a small difference with the increase of (Fr1) for the acute notches (45°,60°), whereas for obtuse notch (90°), E\% was decreased significantly when (Fr1) was increased. Figure (10) shows that when (Fr1) increases, the (E\%) increases linearly for both angles of lower triangle of compound notches. A higher value of (E\%) can be obtained at the small V-notch angle of (120°), while for the compound notches of rectangular and trapezoidal, (E\%) changes are not valuable for a small range of (6.5≤Fr1≤8.0) for a both studied angles.

![Energy Dissipation Ratio (E\%) for Standard Notches](image1)

Figure (9): Relation between energy dissipation ratios (E\%) and (Fr1) for standard notches

![Energy Dissipation Ratio (E\%) for Compound Notches](image2)

Figure (10): Relation between energy dissipation ratios (E\%) and (Fr1) for compound notches

4-3 Reduction of Hydraulic Jump Length (Lj/Y1):

Figure (11) shows the relation between the length of hydraulic jump to the initial depth of flow ratio (Lj/Y1) and (Fr1) for standard notches, as can be seen from this figure (Lj/Y1) greatly depended on (Fr1). For small Froude numbers the amount of (Lj/Y1) is
low for all V-notch angles, while large value of Froude numbers (Fr > 6.0) showed linear variation for the rectangular notch, and higher value of (L_j/Y_1) can be obtained from (0=60°) V-notch. Figure (12) reveals that when (Fr_1) increases, the (L_j/Y_1) decreases for both angles of compound notch of rectangular and triangle, a low value of (L_j/Y_1) can be obtained at the small V-notch angle of (120°), while for compound notch of rectangular and trapezoidal with (0=60°), produced rapidly increase of (L_j/Y_1) as (Fr_1) increased.

Figure (11): Relation between hydraulic jump length ratios (L_j/Y_1) and (Fr_1) for standard notches

Figure (12): Relation between hydraulic jump length ratios (L_j/Y_1) and (Fr_1) for compound notches

4-4 Aeration Efficiency of Compound of regular notch Weirs (AE%):

Figures (13, and 14) show the variation in the self-aeration efficiency (AE %) of standard notches with Froude number (Fr_1). Although the change in discharge is constant, experiments with rectangular and triangular notches indicate that the discharge is the most important factor influencing on the self-aeration efficiency. The aeration efficiency varies with (Fr_1) according to the weir angle. Figure (13) shows significant increase with (AE %) as (Fr_1) increased for V-notch angles of (45°, 60°), a higher value of (AE %) can
be reached at the angle of (60°). A notch of (θ=90°) shows an increase of (AE%) for small values Froude number (Fr₁ ≤ 6.5), but for larger values of (Fr₁), the values of (AE %) decreases. Figure (14) shows relation between (AE %) and (Fr₁) for compound notches. The (120°) compound triangular and rectangular notch shows an increase of (AE %) with increased values of (Fr₁) values, while at low values of (Fr₁), the (130°) compound notch produced the minimum values of the (AE %). No significant changes of the self-aeration efficiency (AE %) produced by compound notches of rectangular and trapezoidal for all studied angles.

Figure (13): Relation between aeration efficiency (AE %) and (Fr₁) for standard notches

Figure (14): Relation between aeration efficiency (AE %) and (Fr₁) for compound notches

The experimental results and discussion of the present work showed that the 45° V-notch produces higher energy dissipation percentage. For 120° compound rectangular and triangular notch produced suitable hydraulic performance in which E% increased up to (35.861%) and AE% (33.53%), therefore the mathematical models derived in this study will be for these types.
5- Dimensional Analysis

Based on the open channel flow theory, the important variables among many factors affecting the coefficient of discharge ($C_d$), the hydraulic jump pattern or length ratio ($L_j/Y_i$) and energy dissipation ratio ($E\%$) are $Y_1,Y_2,P_w, \theta,V_1,V_2,L_j,E_1,E_2,g,\rho,\mu$. The functional relationship for $C_d$, $E\%$, $L_j/Y_i$, and $AE\%$ relating the main flow parameters may be expressed as follows:

$$f_j(Y_1,Y_2,H_j,P_w,\theta,V_1,V_2,L_j,E_1,E_2,g,\rho,\mu) = 0$$

----- (16)

According to Buckingham’s $\pi$-theorem and considering $(Y_1,g,\rho)$ as repeating variables (Gupta et al., 2013), the following dimensionless groups developed:

$$\phi \left( \frac{Y_2}{Y_1}, \frac{H_j}{Y_1}, \frac{L_j}{Y_1}, \frac{V_2}{V_1}, \frac{E_1}{Y_1}, \frac{E_2}{Y_1}, \frac{V_1^2}{gY_1}, \frac{\rho V_1 Y_1}{\mu}, \tan \frac{\theta}{2} \right) = 0$$

----- (17)

From the present study and based on dimensional analysis it is found that almost all the hydraulic characteristics are the function of Froude number $(Fr_i)$, Reynolds number $(Re_i)$, and notch angle $(\theta)$.

$$C_d = f_2 \left( Fr_i, \frac{Y_1}{P_w}, \tan \frac{\theta}{2} \right)$$

----- (18)

$$E\% = f_3 \left( Fr_i, Re_i, \frac{Y_2}{Y_1}, \tan \frac{\theta}{2} \right)$$

----- (19)

$$\frac{L_j}{Y_1} = f_4 \left( Fr_i, Re_i, \frac{H_j}{Y_2}, \tan \frac{\theta}{2} \right)$$

----- (20)

Mathematical models were developed by multiple nonlinear regression analysis for experimental data using (IBM SPSS Statistics 20.0), the performance of the models developed in this work was evaluated based on standard statistical criteria. The statistical measures adopted in this study were the multiple coefficient of determination, and standard error of estimate denoted $R^2$ and SEE respectively. From the basis of nonlinear fitting between different hydraulic characteristics and dimensionless group developed herein, the following models were obtained for:

5-1 Standard V-Notch for $30^\circ \leq \theta \leq 90^\circ$:

$$C_d = 0.651 (Fr_i)^{0.27} \left( \frac{Y_1}{P_w} \right)^{0.539} \left( \tan \frac{\theta}{2} \right)^{0.321} \left[ R^2=0.829; \text{SEE}=0.0773 \right]$$

----- (21)

$$E\% = 25.519 (Fr_i)^{0.659} (Re_i)^{-0.184} \left( \frac{Y_1}{Y_i} \right)^{-0.739} \left( \tan \frac{\theta}{2} \right)^{-0.408} \left[ R^2=0.987; \text{SEE}=1.299 \right]$$

----- (22)

$$\frac{L_j}{Y_1} = 9.128 \times 10^{-4} (Fr_i)^{-4.554} (Re_i)^{2.738} \left( \frac{H_j}{Y_2} \right)^{1.144} \left( \tan \frac{\theta}{2} \right)^{3.738} \left[ R^2=0.876; \text{SEE}=1.437 \right]$$

----- (23)

5-2 Notches Compound of (Rectangle and Triangle) for $\theta=120^\circ$, $130^\circ$:

$$C_d = 0.795 (Fr_i)^{0.17} \left( \frac{Y_1}{P_w} \right)^{-1.1347} \left( \tan \frac{\theta}{2} \right)^{-1.895} \left[ R^2=0.994; \text{SEE}=0.011 \right]$$

----- (24)

$$E\% = 3.047 \times 10^{-8} (Fr_i)^{2.056} (Re_i)^{2.188} \left( \frac{Y_2}{Y_i} \right)^{-0.420} \left( \tan \frac{\theta}{2} \right)^{-0.294} \left[ R^2=0.984; \text{SEE}=5.188 \right]$$

----- (25)

$$\frac{L_j}{Y_1} = -0.606 (Fr_i) + 9.762 (Re_i) + 27.432 \left( \frac{H_j}{Y_2} \right) - 4.159 \left( \tan \frac{\theta}{2} \right) \left[ R^2=0.981; \text{SEE}=0.4627 \right]$$

----- (26)
6-Validation for the Proposed Empirical Models:

The present model equations (21, and 24) for coefficient of discharge are validated using the plot of predicted value of Cd with experimental values of Cd for both V- notch and compound of rectangle and triangle, as shown in figures (15, and 16) respectively.

![Figure (15): Validation of predicted and experimental values of coefficient of discharge for standard V-notches](image1)

![Figure (16): Validation of predicted and experimental values of coefficient of discharge for compound rectangle and triangle notches](image2)

All data points lying within ±10 percentage of the best-fit curves, which reveals good fitting of present model equations.

The present model equations (22, and 25) for energy dissipation ratio (E %) are validated using the plot of predicted value of (E %) with experimental values for both V-notch and compound of rectangle and triangle, as shown in figures (17, and 18) respectively.
Figure (17): Validation of predicted and experimental values of energy dissipation ratio for standard V-notches

All data points lying within ±10 percentage of the best-fit curves, which reveals excellent fitting of present model equations.

The present model equations (23, and 26) for hydraulic jump length ratio are validated using the plot of predicted value of \((L_j/Y_1)\) with experimental values of \((L_j/Y_1)\) for both V-notch and compound of rectangle and triangle, as shown in figures (19, and 20) respectively.
Figure (19): Validation of predicted and experimental values for hydraulic jump length ratio for standard notches

Figure (20): Validation of predicted and experimental values for hydraulic jump length ratio for compound notches

Most of data points lying within ±10 percentage of the best-fit curves, which shows fair fitting of present model equations.

7- Inferences

The conclusions reached based on results obtained are:

[A] For the standard notches:
1-The percentage of energy dissipation ratio decreased with increasing angle of V-notch for all (Fr1) and (Fr2). The V-notch models have given higher percentage to dissipate the flow energy than the rectangle notch.
2-Increasing the angle of V-notches produces flow turbulence, which causing an increase of roller length of the hydraulic jump and the growing of flow turbulence. Consequently, the flow energy dissipation reduced.

[B] For compound regular notches:
1- Less energy dissipation by increasing angle of lower triangle of compound rectangular and triangle notch, as well as the length of each of the hydraulic jump and roller. The hydraulic jump type reflected the turbulence in the flow, which is increasing
with the decrease in the angle of lower notch. The water retention rate of air in the first case is greater than the second case.

2- Less energy dissipation significantly with the increase in side slopes of compound notch of rectangular and trapezoidal, but the decrease in the length of the hydraulic jump with side slope be less. The variation between \((F_{r1})\) and \((F_{r2})\) side slopes would be very small and that reflected directly on the uniformity of the hydraulic jump and stability of the flow. The water retention rate of air to be close for both weir models.

3- Finally, and depending on the percentage of energy dissipation ratio of flow for all models tested in the present work, the V-notch of \((\theta=45^\circ)\) can be inferred that gives the higher percentage to dissipate the energy of flow, but less aeration of hydraulic jump. Whereas according to general hydraulic Performance, the compound of rectangular and triangular notch of \((\theta=120^\circ)\) was adopted as suitable hydraulic model which resulting \((E\%=35.861\%)\) and \((AE\%=33.53\%)\).

References


